Одеський національний університет імені І. І. Мечникова
Факультет математики, фізики та інформаційних технологій
Кафедра методів математичної фізики

Кваліфікаційна робота

на здобуття ступеня вищої освіти «магістр»

«Антиплоська задача теорії пружності для шаруватої прямокутної області з міжфазними дефектами»

«Anti-plane problem of the theory of elasticity for the multilayered rectangular region with interfacial defects»

Виконав: здобувач денної форми навчання
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Чернобровкін Артем Валерійович

Керівник: канд. фіз.-мат. наук, доц. Журавльова З.Ю. ____
Рецензент: канд. фіз.-мат. наук, доц. Процеров Ю.С.

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Odesa I. I. Mechnikov National University
Faculty of Mathematics, Physics and Information Technology
Department of methods of mathematical physics

Diploma thesis

master

«Antiplane problem of the theory of elasticity for multilayered rectangular region with interfacial defects»

Fulfilled by: full-time student
specialty 113 Applied Mathematics
Artem Chernobrovkin

Supervisor: Ph. D. in physics and mathematics sciences, Associate Professor Zinaida Zhuravlova
Reviewer: Ph. D. in physics and mathematics sciences, Associate Professor Yuriy Protserov

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CONTENTS

ВСТУП

Актуальність.Інженерні проблеми визначення міцності конструкцій потребують адекватних та простих математичних моделей, за допомогою яких можна підраховувати величини напружень та їх розподіл усередині області.

Реальні конструкції досить часто складаються декількох шарів заради зміцнення або ізолювання деяких речовин між собою. Також слід зауважити, що особим класом задач є задачі з різними дефектами. Зокрема, виокремлюють два види дефектів: тріщина або включення.

Слід зауважити, що часто виникають ситуації, коли впродовж тривалого часу на конструкціях такого типу можуть з'явитись тріщини. Такого роду дефекти можуть виникти під впливом різних природних умов: вітру, води чи механічної дії.

Данна робота складається з чотирьох розділів, в яких було розв'язано такі модельні задачі:

1) В першому та другому розділі розглянута прямокутна область, що складається з N-шарів та окремий випадок, коли $N = 3$.
2) В третьому та четвертому розділі розглянута прямокутна область, що складається з N-шарів та окремий випадок, коли $N = 3$, яка послаблена на місці стику тріщинами.

Всі ці задачі було розв'язано за допомогою методу інтегральних перетворень. Використання рекурентних співвідношень для визначення сталих одного шару через сталі іншого дозволяє будувати розв'язок для багатошарової області будь-якої складності.

Також слід відзначити, що у випадку дефекту треба побудувати сінгуляне інтегро-диференціальне рівняння.

Мета.Для багатошарової області, на прикладі трьошарової, прямокутної області досліджено розподіл напружень у залежності від співвідношення модулів пружності шарів та геометричних параметрів області.

Також розглянуто різні види навантажень та характеристик матеріалу у дефекті.

Обчислити коефіцієнти інтенсивності напружень в залежності від довжини тріщини.

Об'єкт дослідження. N-шарова прямокутна область, що знаходиться у стані антиплоської деформаії та N-шарова прямокутна область, що знаходиться у стані антиплоської деформаії, що послаблена міжфазними тріщинами.

INTRODUCTION

Relevance. The engineering problems of determining the strength of structures require adequate and simple mathematical models which can be used to calculate the values of loads and their distribution in the middle of an area.

Simple constructions are often made of several layers to cement or isolation of certain substances among themselves. It should also be noted that a particular class of problems has different defects. In particular, two types of defects are identified: fracture or inclusion.

It should be noted that there are often situations where cracks can appear on structures of this type over a long period. The defects can occur under various environmental conditions: wind, water, and sneezing.

This work consists of four sections in which these model problems were solved:

1) In the first and second sections, a rectangular area consists of N-layered and a single case where $N = 3$ are considered.
2) In the third and fourth sections, a rectangular area consists of N-layered in a single case where $N = 3$ is considered, which is relaxed at the sticking point by frictions.

All these problems have been solved using the method of integral transformations. Using recurrent relations to determine the steels of one layer through the steels of another allows one to construct solutions for any complexity of different difficulties.

It should also be noted that a singular integral referential equation must be created in the case of a defect.

Goal. For a three-layer, rectangular area, the distribution of loads depending on the ratio of the layer twist's modulus and the area's geometrical parameters is investigated.

Also, different kinds of stresses and characteristics of the material in the defect are considered.

Calculate the coefficients of the loads' intensity depending on the fracture's length.

Subject of study. N- a layered rectangular region in a state of antiplane deformation and N-a layers, a rectangular area in a state of antiplane deformation loosened by infernal cracks.

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Calculate the coefficients of the loads' intensity depending on the fracture's length.

Subject of study. N-th layers, rectangular region in a state of antiplane deformation and N-a layers, a rectangular area in a state of antiplane deformation loosened by infernal cracks.

CHAPTER 1

THE ANTI-PLANE PROBLEM OF ELASTICITY THEORY FOR MULTILAYERED RECTANGULAR AREA

1.1. Statement of the problem

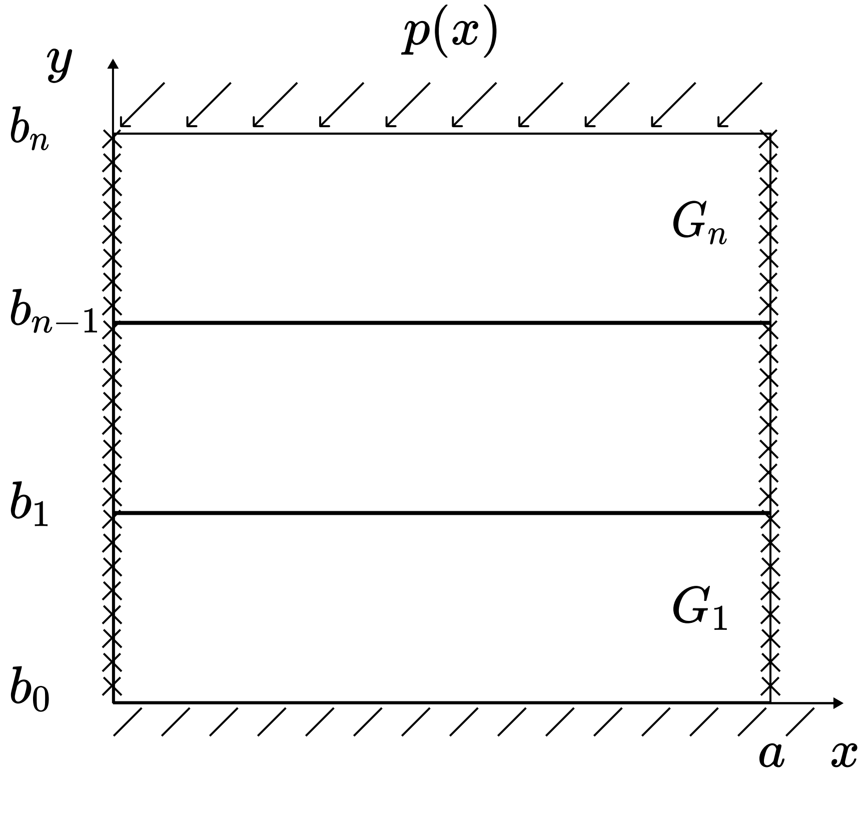


Figure 1.1. Geometry and coordinate system of a rectangular area

The area under consideration (Fig.1.1) (G_k — shear modulus of the k-th layer) occupies the area described in the Cartesian coordinate system by the relations $0 < x < a, b_{k-1} < y < b_k$, which is in a state of anti-plane deformation. This region is divided into N heterogeneous layers along the y axis. Let the edges $x = 0$,$x = a$ be immovably fixed.

$$W_k\Big|_{x=0} = 0, \ W_k\Big|_{x=a} = 0, \ b_{k-1} < y < b_k, \ \overline{1,N} \tag{1.1}$$

where $W_k\left(x,y\right)$— movement relative to the z axis in the k-th layer, $b_0 = 0, b_N = b$. The face $y = 0$ is in smooth contact conditions, the face $y = b$ is subjected to a load of intensity $p\left(x\right)$

$$\left.\tau_{yz}^{1}\right|_{y=0} = 0,\ \left.\tau_{yz}^{N}\right|_{y=b_N} = p\left(x\right), 0 < x < a \tag{1.2}$$

where $\tau_{yz}^{1}\left(x,y\right), \tau_{yz}^{N}\left(x,y\right)$— the tangential stresses of the first and N-layers, respectively.
The conjugation conditions are fulfilled between the layers:

$$\begin{aligned} &\left.W_k\right|_{y=b_k-0} = \left.W_{k+1}\right|_{y=b_k+0}, \\ &\qquad\qquad\qquad\qquad 0 < x < a, k = \overline{1,N-1} \\ &\left.\tau_{yz}^{k}\right|_{y=b_k-0} = \left.\tau_{yz}^{k+1}\right|_{y=b_k+0} \end{aligned} \tag{1.3}$$

It is necessary to find the displacement and stress of each of the layers satisfying the conditions (1.1)— (1.3) and the equilibrium equation

$$\frac{\partial^2 W_k}{\partial x^2} + \frac{\partial^2 W_k}{\partial y^2} = 0,\ 0 < x < a,\ b_{k-1} < y < b_k \tag{1.4}$$

Write down the boundary value problem

$$\begin{cases} \dfrac{\partial^2 W_k}{\partial x^2} + \dfrac{\partial^2 W_k}{\partial y^2} = 0, & 0 < x < a, \qquad b_{k-1} < y < b_k \\ \left.W_k\right|_{x=0} = 0, \qquad \left.W_k\right|_{x=a} = 0 & b_{k-1} < y < b_k \\ \left.\tau_{yz}^{1}\right|_{y=0} = 0, \qquad \left.\tau_{yz}^{N}\right|_{y=b_N} = p\left(x\right), & 0 < x < a, \qquad b_{k-1} < y < b_k \end{cases} \tag{1.5}$$

where:

$$\tau_{yz}^{1}\Big|_{y=0} = 0 \Leftrightarrow \frac{\partial W_1}{\partial y}\Big|_{y=0} = 0, \quad \tau_{yz}^{N}\Big|_{y=b_N} = p(x) \Leftrightarrow \frac{\partial W_N}{\partial y}\Big|_{y=b_N} = \frac{p(x)}{G_N} \quad (1.6)$$

Conjugation conditions:

$$\begin{aligned} W_k\Big|_{y = b_k - 0} &= W_{k+1}\Big|_{y = b_k + 0}, \\ &\qquad\qquad 0 < x < a, k = \overline{1,N-1} \\ \tau_{yz}^{k}\Big|_{y = b_k - 0} &= \tau_{yz}^{k+1}\Big|_{y = b_k + 0} \end{aligned} \quad (1.7)$$

Where:

$$\tau_{yz}^{k}\Big|_{y=b_k-0} = \tau_{yz}^{k+1}\Big|_{y=b_k+0} \Leftrightarrow G_k \frac{\partial W_k}{\partial y}\Big|_{y=b_k-0} = G_{k+1}\frac{\partial W_{k+1}}{\partial y}\Big|_{y=b_k+0}$$

G_k — shear modulus of the k-th layer, $k = \overline{1,N-1}$

1.2. Reducing the initial problem to the one-dimensional problem

We reduce the input problem to a one-dimensional one using the finite integral sin Fourier transform of the variable x:

$$W_{\alpha_n,k}(y) = \int_0^a W_k(x,y)\sin\alpha_n x \ \ \mathrm{d}x \quad (1.8)$$

With the inversion formula:

$$W_k(x,y) = \frac{2}{a}\sum_{n=0}^{\infty} W_{\alpha_n,k}(y)\sin\alpha_n x \quad (1.9)$$

Write the boundary conditions in the transform domain:

$$W'_{\alpha_n,1}\Big|_{y=0} = \int_0^a \frac{\partial W_1}{\partial y}\Big|_{y=0} \sin\alpha_n x \ \ \mathrm{d}x \tag{1.10}$$

$$W'_{\alpha_n,N}\Big|_{y=b_N} = \int_0^a \frac{\partial W_N}{\partial y}\Big|_{y=b_N} \sin\alpha_n x \ \ \mathrm{d}x \tag{1.11}$$

Write the load in the transform domain:

$$\frac{p_{\alpha_n}}{G_N} = \int_0^a \frac{p(x)}{G_N} \sin\alpha_n x \ \ \mathrm{d}x \tag{1.12}$$

Write down the conjugation conditions in the transform domain:

$$W_{\alpha_n,k}\Big|_{y=b_k-0} = \int_0^a W_k\Big|_{y=b_k-0} \sin\alpha_n x \ \ \mathrm{d}x \tag{1.13}$$

$$W_{\alpha_n,k+1}\Big|_{y=b_k+0} = \int_0^a W_{k+1}\Big|_{y=b_k+0} \sin\alpha_n x \ \ \mathrm{d}x \tag{1.14}$$

$$G_k W'_{\alpha_n,k}\Big|_{y=b_k-0} = G_k \int_0^a \frac{\partial W_k}{\partial y}\Big|_{y=b_k-0} \sin\alpha_n x \ \ \mathrm{d}x \tag{1.15}$$

$$G_{k+1} W'_{\alpha_n,2}\Big|_{y=b_k+0} = G_{k+1} \int_0^a \frac{\partial W_{k+1}}{\partial y}\Big|_{y=b_k+0} \sin\alpha_n x \ \ \mathrm{d}x \tag{1.16}$$

Write down the boundary value problem:

$$\begin{cases} W''_{\alpha_n,k}(y) - \alpha_n^2 W_{\alpha_n,k}(y) = 0 \\ W'_{\alpha_n,1}\Big|_{y=0} = 0, \quad W'_{\alpha_n,N}\Big|_{y=b_N} = \dfrac{p_{\alpha_n}}{G_N} \\ W_{\alpha_n,k}\Big|_{y=b_k-0} = W_{\alpha_n,k+1}\Big|_{y=b_k+0} \\ W'_{\alpha_n,k}\Big|_{y=b_1-0} = \dfrac{G_{k+1}}{G_k} W'_{\alpha_n,k+1}\Big|_{y=b_k+0} \end{cases} \tag{1.17}$$

where α_n — integral transform parameter, G_N — shear modulus of the k-th layer

The general solutions (1.17) will then be found in the form:

$$\begin{aligned} W_{\alpha_n,k}(y) &= A_k e^{\alpha_n y} + B_k e^{-\alpha_n y} \\ W'_{\alpha_n,k}(y) &= \alpha_n \left(A_k e^{\alpha_n y} - B_k e^{-\alpha_n y}\right) \end{aligned} \tag{1.18}$$

where A_k,B_k — unknown constants.
Using conjugation conditions:

$$\begin{cases} A_k e^{\alpha_n b_k} + B_k e^{-\alpha_n b_k} = A_{k+1} e^{\alpha_n b_k} + B_{k+1} e^{-\alpha_n b_k} \\ G_k (A_k e^{\alpha_n b_k} - B_k e^{-\alpha_n b_k}) = G_{k+1} \left(A_{k+1} e^{\alpha_n b_k} - B_{k+1} e^{-\alpha_n b_k}\right) \end{cases} \tag{1.19}$$

Build a matrix:

$$H_k(y) = \begin{pmatrix} e^{\alpha_n y} & e^{-\alpha_n y} \\ G_k e^{\alpha_n y} & -G_k e^{-\alpha_n y} \end{pmatrix} \tag{1.20}$$

From the conjugation conditions (1.19), we express A_k, B_k in terms of A_1, B_1 [2] by the recurrent formula:

$$\begin{pmatrix} A_k \\ B_k \end{pmatrix} = P_k \begin{pmatrix} A_1 \\ B_1 \end{pmatrix}$$

where $P_k = H_k^{-1}(b_{k-1}) \cdot H_{k-1}(b_{k-1}) \cdot \dots \cdot H_2^{-1}(b_1) H_1(b_1)$
The unknown constance of the first layer are found from the boundary conditions of the problem (1.17).
As a result, analytical representations of movements $W_{\alpha_n,k}$in transform domain were obtained.
In this work, the case when $N = 3$ is considered.

CHAPTER 2

THE ANTI-PLANE PROBLEM OF THE THEORY OF ELASTICITY FOR A THREE-LAYER RECTANGULAR REGION

2.1. Reducing the initial problem to the one-dimensional problem

From the conjugation conditions (1.19), A_2, B_2,A_3, B_3were expressed in terms of A_1, B_1 [2]:

$$A_2 = \frac{A_1}{2} + \frac{A_1}{2G_{21}} + \frac{B_1 e^{-2\alpha_n b_1}}{2} - \frac{B_1 e^{-2\alpha_n b_1}}{2G_{21}}$$

$$A_3 = \frac{A_1 e^{2\alpha_n b_1} e^{-2\alpha_n b_2}}{4} + \frac{A_1}{4} - \frac{A_1 e^{2\alpha_n b_1} e^{-2\alpha_n b_2}}{4G_{32}} + \frac{A_1}{4G_{32}} - \frac{A_1 e^{2\alpha_n b_1} e^{-2\alpha_n b_2}}{4G_{21}} +$$
$$+ \frac{A_1}{4G_{21}} + \frac{A_1 e^{2\alpha_n b_1} e^{-2\alpha_n b_2}}{4G_{21}G_{32}} + \frac{A_1}{4G_{21}G_{32}} + \frac{B_1 e^{-2\alpha_n b_2}}{4} + \frac{B_1 e^{-2\alpha_n b_1}}{4} - \frac{B_1 e^{-2\alpha_n b_2}}{4G_{32}} +$$
$$+ \frac{B_1 e^{-2\alpha_n b_1}}{4G_{32}} + \frac{B_1 e^{-2\alpha_n b_2}}{4G_{21}} - \frac{B_1 e^{-2\alpha_n b_1}}{4G_{21}} - \frac{B_1 e^{-2\alpha_n b_2}}{4G_{21}G_{32}} - \frac{B_1 e^{-2\alpha_n b_1}}{4G_{21}G_{32}}$$

$$B_2 = \frac{A_1 e^{2\alpha_n b_1}}{2} - \frac{A_1 e^{2\alpha_n b_1}}{2G_{21}} + \frac{B_1}{2} + \frac{B_1}{2G_{21}}$$

$$B_3 = \frac{A_1 e^{2\alpha_n b_1}}{4} + \frac{A_1 e^{2\alpha_n b_2}}{4} + \frac{A_1 e^{2\alpha_n b_1}}{4G_{32}} - \frac{A_1 e^{2\alpha_n b_2}}{4G_{32}} - \frac{A_1 e^{2\alpha_n b_1}}{4G_{21}} + \frac{A_1 e^{2\alpha_n b_2}}{4G_{21}} -$$
$$- \frac{A_1 e^{2\alpha_n b_1}}{4G_{21}G_{32}} - \frac{A_1 e^{2\alpha_n b_2}}{4G_{21}G_{32}} + \frac{B_1}{4} + \frac{B_1 e^{-2\alpha_n b_1} e^{2\alpha_n b_2}}{4} + \frac{B_1}{4G_{32}} - \frac{B_1 e^{-2\alpha_n b_1} e^{2\alpha_n b_2}}{4G_{32}} + \frac{B_1}{4G_{21}} -$$
$$- \frac{B_1 e^{-2\alpha_n b_1} e^{2\alpha_n b_2}}{4G_{21}} + \frac{B_1}{4G_{21}G_{32}} + \frac{B_1 e^{-2\alpha_n b_1} e^{2\alpha_n b_2}}{4G_{21}G_{32}}$$

The unknown steels of the first layer are found from the boundary conditions of the problem (1.17).
the following:

$$A_1 = \frac{4G_{21}G_{32}p_{\alpha_n}e^{\alpha_n b}e^{2\alpha_n b_1}e^{2\alpha_n b_2}}{G_3\alpha_n\left(G_{21}\left(G_{32}(u_8 - e^{4\alpha_n b_2}) + u_0e^{2\alpha_n b_2} + u_{10}\right) + G_{32}(u_9 + e^{4\alpha_n b_2}) + u_1e^{2\alpha_n b_2} + u_{11}\right)}$$

$$B_1 = \frac{4G_{21}G_{32}p_{\alpha_n}e^{\alpha_n b}e^{2\alpha_n b_1}e^{2\alpha_n b_2}}{G_3\alpha_n\left(G_{21}\left(G_{32}u_{24} + u_{25}\right) + G_{32}u_{23} + u_{26}\right)}$$

$$u_0 = e^{2\alpha_n b} - e^{4\alpha_n b_1}$$
$$u_1 = -e^{2\alpha_n b} + e^{4\alpha_n b_1}$$
$$u_2 = e^{2\alpha_n b_1} + e^{2\alpha_n b_2} + 1$$
$$u_3 = -e^{2\alpha_n b_1} + e^{2\alpha_n b_2} + 1$$
$$u_4 = -e^{4\alpha_n b_2} - e^{2\alpha_n b_2}$$
$$u_5 = -e^{2\alpha_n b_1} + e^{2\alpha_n b_2} - 1$$
$$u_6 = e^{2\alpha_n b_1} + e^{2\alpha_n b_2} - 1$$
$$u_7 = e^{4\alpha_n b_2} - e^{2\alpha_n b_2}$$
$$u_8 = u_0e^{2\alpha_n b_2} + \left(u_2e^{2\alpha_n b} + u_4\right)e^{2\alpha_n b_1}$$
$$u_9 = u_1e^{2\alpha_n b_2} + \left(u_3e^{2\alpha_n b} + u_4\right)e^{2\alpha_n b_1}$$
$$u_{10} = \left(u_5e^{2\alpha_n b} + u_7\right)e^{2\alpha_n b_1} + e^{4\alpha_n b_2}$$
$$u_{11} = \left(u_6e^{2\alpha_n b} + u_7\right)e^{2\alpha_n b_1} - e^{4\alpha_n b_2}$$
$$u_{12} = \left(e^{2\alpha_n b_2} + 1\right)$$
$$u_{13} = \left(u_{12}e^{2\alpha_n b_1} + e^{4\alpha_n b_1} + e^{2\alpha_n b_2}\right)$$
$$u_{14} = -e^{4\alpha_n b_1}e^{2\alpha_n b_2} - e^{4\alpha_n b_2}$$
$$u_{15} = e^{4\alpha_n b_2} - e^{2\alpha_n b_2}$$
$$u_{16} = e^{2\alpha_n b_2} - 1$$
$$u_{17} = u_{16}e^{2\alpha_n b_1} - e^{4\alpha_n b_1} + e^{2\alpha_n b_2}$$
$$u_{18} = -e^{4\alpha_n b_1}e^{2\alpha_n b_2} + e^{4\alpha_n b_2}$$
$$u_{19} = u_{12}e^{2\alpha_n b_1} - e^{4\alpha_n b_1} - e^{2\alpha_n b_2}$$
$$u_{20} = u_{12}e^{2\alpha_n b_1} - e^{4\alpha_n b_1} - e^{2\alpha_n b_2}$$
$$u_{21} = e^{4\alpha_n b_1}e^{2\alpha_n b_2} + e^{4\alpha_n b_2}$$
$$u_{22} = e^{4\alpha_n b_1}e^{2\alpha_n b_2} - e^{4\alpha_n b_2}$$
$$u_{23} = u_4e^{2\alpha_n b_1} + u_{19}e^{2\alpha_n b} + u_{21}$$
$$u_{24} = u_4e^{2\alpha_n b_1} + u_{13}e^{2\alpha_n b} + u_{14}$$
$$u_{25} = u_{15}e^{2\alpha_n b_1} + u_{17}e^{2\alpha_n b} + u_{18}$$
$$u_{26} = u_{15}e^{2\alpha_n b_1} + u_{20}e^{2\alpha_n b} + u_{22}$$

Substitute A_1,B_1,A_2,B_2,A_3,B_3 in W_k,$W_k^{'}$,$(k = \overline{1,3})$ from the system (1.17) and using the boundary conditions of the system (1.17). A solution was obtained in the transform domain:

$$W_1 = \frac{4G_{21}G_{32}p_{\alpha_n}T_1}{G_3\alpha_n\left(\left(-1 + e^{-2\alpha_n b}\right)\left(R_3 - 1\right) + S\right)} \tag{2.1}$$

$$W_2 = \frac{2G_{32}p_{\alpha_n}\left(\left(G_{21} - 1\right)T_2 + \left(G_{21} + 1\right)T_1\right)}{G_3\alpha_n\left(\left(-1 + e^{-2\alpha_n b}\right)\left(R_3 - 1\right) + S\right)} \tag{2.2}$$

$$W_3 = \frac{p_{\alpha_n}\left(T_1\left(F_1 + 1\right) + T_2\left(F_2 - 1\right) + T_3\left(F_3 - 1\right) + T_4\left(R_5 + 1\right)\right)}{G_3\alpha_n\left(\left(-1 + e^{-2\alpha_n b}\right)\left(R_3 - 1\right) + S\right)} \tag{2.3}$$

where:

$$E_1 = e^{-2\alpha_n(b-b_2)} - e^{-2\alpha_n b_2} \qquad T_1 = e^{-\alpha_n(b+y)} + e^{-\alpha_n(b-y)}$$
$$E_2 = e^{-2\alpha_n(b-b_1)} - e^{-2\alpha_n b_1} \qquad T_2 = e^{-\alpha_n(b+2b_1-y)} + e^{-\alpha_n(b-2b_1+y)}$$
$$E_3 = e^{-2\alpha_n(b+b_1-b_2)} \qquad T_3 - e^{-\alpha_n(b-2b_2+y)} + e^{-\alpha_n(b+2b_1-y)}$$
$$E_4 = e^{-2\alpha_n(-b_1+b_2)}$$

$$T_4 = e^{-\alpha_n(b+2b_1-2b_2+y)} + e^{-\alpha_n(b-2b_1+2b_2-y)}$$

$$F_1 = G_{21}G_{32} + G_{21} + G_{32} \qquad R_2 = -G_{21}G_{32} - G_{21} + G_{32}$$
$$F_2 = G_{21}G_{32} + G_{21} - G_{32} \qquad R_3 = -G_{21}G_{32} - G_{21} - G_{32}$$
$$F_3 = G_{21}G_{32} - G_{21} + G_{32} \qquad R_4 = -G_{21}G_{32} + G_{21} + G_{32}$$
$$R_1 = -G_{21}G_{32} + G_{21} - G_{32} \qquad R_5 = G_{21}G_{32} - G_{21} - G_{32}$$

$$S = E_2(R_2+1) + E_1(R_1+1) + (R_4-1)E_3 + (R_5+1)E_4$$

2.2. Inversion of integral transforms

The found solutions in the transform domain are inverted according to the formula (1.9). It is derived:

$$W_1(x,y) = \frac{2}{a}\sum_{n=0}^{\infty} \frac{4G_{21}G_{32}p_{\alpha_n}T_1 \sin\alpha_n x}{G_3\alpha_n\left(\left(-1+e^{-2\alpha_n b}\right)(R_3-1)+S\right)} \tag{2.4}$$

$$W_2(x,y) = \frac{2}{a}\sum_{n=0}^{\infty} \frac{2G_{32}p_{\alpha_n}\left((G_{21}-1)T_2 + (G_{21}+1)T_1\right)\sin\alpha_n x}{G_3\alpha_n\left(\left(-1+e^{-2\alpha_n b}\right)(R_3-1)+S\right)} \tag{2.5}$$

$$W_3(x,y) = \frac{2}{a}\sum_{n=0}^{\infty} \frac{p_{\alpha_n}\left(T_1(F_1+1) + T_2(F_2-1) + T_3(F_3-1) + T_4(R_5+1)\right)}{G_3\alpha_n\left(\left(-1+e^{-2\alpha_n b}\right)(R_3-1)+S\right)} \tag{2.6}$$

Given (1.12), formulas (2.4),(2.5) and (2.6) can be rewritten in the following form:

$$W_1(x,y) = \frac{2}{a}\int_0^a \sum_{n=0}^{\infty} \frac{4G_{21}G_{32}p(\xi)T_1\Omega(x,\xi)}{G_3\alpha_n\left(\left(-1+e^{-2\alpha_n b}\right)(R_3-1)+S\right)}\,\mathrm{d}\xi \tag{2.7}$$

$$W_2\,(x,y) = \frac{2}{a}\int_0^a \sum_{n=0}^{\infty} \frac{2G_{32}p(\xi)\,((G_{21}-1)\,T_2 + (G_{21}+1)\,T_1)\,\Omega(x,\xi)}{G_3\alpha_n\,((-1+e^{-2\alpha_n b})\,(R_3-1)+S)}\ \mathrm{d}\xi \quad (2.8)$$

$$W_3\,(x,y) =$$
$$\frac{2}{a}\int_0^a \sum_{n=0}^{\infty} \frac{p_{\alpha_n}\,(T_1\,(F_1+1) + T_2\,(F_2-1) + T_3\,(F_3-1) + T_4\,(R_5+1))}{G_3\alpha_n\,((-1+e^{-2\alpha_n b})\,(R_3-1)+S)}\ \mathrm{d}\xi$$
(2.9)

where: $\Omega(x,\xi) = \sin\alpha_n x \cdot \sin\alpha_n \xi$

2.3. Checking of the boundary conditions

It should be noted that no difficulties arose during the verification of homogeneous boundary conditions, so the case of non-homogeneous boundary conditions for moving W_2 was considered.

$$\left.\frac{\partial W_3}{\partial y}\right|_{y=b} = \frac{p(x)}{G_3},$$

де

$$\left.\frac{\partial W_3}{\partial y}\right|_{y=b} = \frac{2}{a}\int_0^a \sum_{n=0}^{\infty} \frac{p(\xi)\sin(\alpha_n x)\sin(\alpha_n \xi)}{G_3}\, d\xi$$

In order to check this inhomogeneous boundary condition, the order of integration and summation should be changed: $\frac{2}{a}\sum_{n=0}^{\infty}\left[\int_0^a p(\xi)\sin(\alpha_n\xi)\,d\xi\right]\sin(\alpha_n x)$

Continue the load $p(x)$ in the original problem in an odd way to check the inhomogeneity of the boundary condition because there is no material for $-a < x < 0$. Moreover, the development in the Fourier series for the expanded $p(x)$ was used. Loads were selected in such a way that $p(0) = 0$; otherwise, there will be a gap of the first kind.

2.4. Summarization of weakly convergent parts of series

Consider the series $\sum_{n=1}^{\infty} a(n)$, which is weakly convergent. For separation of its weakly convergent part, the following technique [3] is used, namely: The series $\sum_{n=1}^{\infty} a(n)$ is split into two terms $\sum_{n=1}^{\infty} a(n) = \sum_{n=0}^{A} a(n) + \sum_{n=A}^{\infty} a(n)$. In the second obtained series, the function is replaced by its asymptotic representation at $n \to \infty$, after which the term $\sum_{n=0}^{A} \tilde{a}(n)$ is added and subtracted, where $\tilde{a}(n)$ is the asymptotic representation of the function $a(n)$. So:

$$\sum_{n=1}^{\infty} a(k) = \sum_{n=0}^{\infty} \tilde{a}(n) + \sum_{n=0}^{A} \left(a(n) - \tilde{a}(n)\right), \;\; A \to \infty \tag{2.10}$$

The series $\sum_{n=0}^{\infty} \tilde{a}(n)$ included in this representation can be summed by using the following [4] formulas:

$$\sum_{n=0}^{\infty} e^{-nt} \sin nx = \frac{1}{2} \frac{\sin x}{\cosh t - \cos x} \tag{2.11}$$

After integration of the formula (2.11). It is derived:

$$\sum_{n=1}^{\infty} \frac{1}{n} e^{-nt} \cos nx = -\frac{1}{2} \ln\left(\operatorname{ch} t - \cos x\right) \tag{2.12}$$

2.4.1. Summary of weakly convergent parts of $W_1(x;y)$

$$W_1\left(x,y\right)=\frac{2}{a}\int_0^a\sum_{n=0}^{\infty}\frac{4G_{21}G_{32}p(\xi)T_1\Omega(x,\xi)}{G_3\alpha_n\left(\left(-1+e^{-2\alpha_n b}\right)\left(R_3-1\right)+S\right)}\ \mathrm{d}\xi$$

where: $T_1=e^{-\alpha_n(b+y)}+e^{-\alpha_n(b-y)}$, $\Omega(x,\xi)=\frac{1}{2}(\cos\alpha_{nn}(\xi-x)-\cos\alpha_n(\xi+x))$
Applying the formula (2.12) it is obtained:

$$\sum_{n=1}^{\infty}\frac{1}{\alpha_n}\cos\alpha_n\left(\xi-x\right)e^{-\alpha_n(b+y)}=-\frac{a}{2\pi}\ln\left(\operatorname{ch}t_0-\cos x_0\right)$$
$$\sum_{n=1}^{\infty}\frac{1}{\alpha_n}\cos\alpha_n\left(\xi-x\right)e^{-\alpha_n(b-y)}=-\frac{a}{2\pi}\ln\left(\operatorname{ch}t_1-\cos x_0\right)$$
$$\sum_{n=1}^{\infty}-\frac{1}{\alpha_n}\cos\alpha_n\left(\xi+x\right)e^{-\alpha_n(b+y)}=\frac{a}{2\pi}\ln\left(\operatorname{ch}t_0-\cos x_1\right)$$
$$\sum_{n=1}^{\infty}-\frac{1}{\alpha_n}\cos\alpha_n\left(\xi+x\right)e^{-\alpha_n(b-y)}=\frac{a}{2\pi}\ln\left(\operatorname{ch}t_1-\cos x_1\right)$$

where:

$$t_0=\frac{\pi}{a}(b+y);\qquad x_0=\frac{\pi}{a}(\xi-x);$$
$$t_1=\frac{\pi}{a}(b-y);\qquad x_1=\frac{\pi}{a}(\xi+x);$$

Then the weakly convergent part for $W_1(x;y)$ will have the form:

$$\sum_{n=0}^{A}\tilde{a}(n)=\sum_{w=1}^{4}\Upsilon_1 C_w$$

here and further:

$$C_1=-\ln\left(\operatorname{ch}t_0-\cos x_0\right)\qquad C_3=\ln\left(\operatorname{ch}t_0-\cos x_1\right)$$
$$C_2=-\ln\left(\operatorname{ch}t_1-\cos x_0\right)\qquad C_4=\ln\left(\operatorname{ch}t_1-\cos x_1\right)$$
$$\Upsilon_1=-\frac{4aG_{21}G_{32}}{4\pi(R_3-1)}$$

Then $W_1(x;y)$ will take the form:

$$W_1(x;y) = \frac{2}{aG_3}\int_0^a p(\xi)\left[\Upsilon_1 \sum_{w=1}^{4} C_w + \frac{1}{2}\sum_{n=1}^{A} \Xi(x,\xi)[Q_1 - Q_2]\right] d\xi \qquad (2.13)$$

here and further:

$$\Xi(x,\xi) = \cos\alpha_{nn}(\xi - x) - \cos\alpha_{nn}(\xi + x)$$
$$Q_1 = \frac{4G_{21}G_{32}T_1}{\alpha_n\left(\left(-1 + e^{-2\alpha_n b}\right)(R_3 - 1) + S\right)}$$
$$Q_2 = -\frac{4G_{21}G_{32}T_1}{\alpha_n(R_3 - 1)}$$

2.4.2. Summary of weakly convergent parts of $\tau_{yz}^1(x;y)$

$$\frac{\partial W_1}{\partial y}(x;y) = \frac{2G_1}{aG_3}\int_0^a p(\xi)\left[\Upsilon_1 \sum_{w=1}^{4} C_w^{'} + \frac{1}{2}\sum_{n=1}^{A} \Xi(x,\xi)[Q_1^{'} - Q_2^{'}]\right] d\xi \qquad (2.14)$$

where:

$$C_1^{'} = -\frac{\pi}{a}\frac{\operatorname{sh} t_0}{(\operatorname{ch} t_0 - \cos x_0)} \qquad C_3^{'} = \frac{\pi}{a}\frac{\operatorname{sh} t_0}{(\operatorname{ch} t_0 - \cos x_1)}$$
$$C_2^{'} = \frac{\pi}{a}\frac{\operatorname{sh} t_1}{(\operatorname{ch} t_1 - \cos x_0)} \qquad C_4^{'} = -\frac{\pi}{a}\frac{\operatorname{sh} t_1}{(\operatorname{ch} t_1 - \cos x_1)}$$
$$Q_1^{'} = \frac{4G_{21}G_{32}T_1^{'}}{\alpha_n\left(\left(-1 + e^{-2\alpha_n b}\right)(R_3 - 1) + S\right)}$$
$$Q_2^{'} = -\frac{4G_{21}G_{32}T_1^{'}}{\alpha_n(R_3 - 1)}$$
$$T_1^{'} = -\alpha_n(-e^{-\alpha_{nn}(b-y)} + e^{-\alpha_{nn}(b+y)})$$

2.4.3. Summary of weakly convergent parts of $W_2(x;y)$

$$W_2\,(x,y) = \frac{2}{a}\int_0^a \sum_{n=0}^{\infty} \frac{2G_{32}p(\xi)\left(\left(G_{21}-1\right)T_2 + \left(G_{21}+1\right)T_1\right)\Omega(x,\xi)}{G_3\alpha_n\left(\left(-1+e^{-2\alpha_n b}\right)\left(R_3-1\right)+S\right)}\ \mathrm{d}\xi$$

where:
$T_1 = e^{-\alpha_n(b+y)} + e^{-\alpha_n(b-y)}$,
$T_2 = e^{-\alpha_n(b+2b_1-y)} + e^{-\alpha_n(b-2b_1+y)}$,
$\Omega(x,\xi) = \frac{1}{2}(\cos\alpha_{nn}(\xi - x) - \cos\alpha_{nn}(\xi + x))$
Applying the formula (2.12) it is obtained:

$$\sum_{n=1}^{\infty}\frac{1}{\alpha_n}\cos\alpha_n\,(\xi - x)e^{-\alpha_n(b+y)} = -\frac{a}{2\pi}\ln\left(\operatorname{ch}t_0 - \cos x_0\right)$$
$$\sum_{n=1}^{\infty}\frac{1}{\alpha_n}\cos\alpha_n\,(\xi - x)e^{-\alpha_n(b-y)} = -\frac{a}{2\pi}\ln\left(\operatorname{ch}t_1 - \cos x_0\right)$$
$$\sum_{n=1}^{\infty}-\frac{1}{\alpha_n}\cos\alpha_n\,(\xi + x)e^{-\alpha_n(b+y)} = \frac{a}{2\pi}\ln\left(\operatorname{ch}t_0 - \cos x_1\right)$$
$$\sum_{n=1}^{\infty}-\frac{1}{\alpha_n}\cos\alpha_n\,(\xi + x)e^{-\alpha_n(b-y)} = \frac{a}{2\pi}\ln\left(\operatorname{ch}t_1 - \cos x_1\right)$$
$$\sum_{n=1}^{\infty}\frac{1}{\alpha_n}\cos\alpha_n\,(\xi - x)e^{-\alpha_n(b+2b_1-y)} = -\frac{a}{2\pi}\ln\left(\operatorname{ch}t_2 - \cos x_0\right)$$
$$\sum_{n=1}^{\infty}\frac{1}{\alpha_n}\cos\alpha_n\,(\xi - x)e^{-\alpha_n(b-2b_1+y)} = -\frac{a}{2\pi}\ln\left(\operatorname{ch}t_3 - \cos x_0\right)$$
$$\sum_{n=1}^{\infty}-\frac{1}{\alpha_n}\cos\alpha_n\,(\xi + x)e^{-\alpha_n(b+2b_1-y)} = \frac{a}{2\pi}\ln\left(\operatorname{ch}t_2 - \cos x_1\right)$$
$$\sum_{n=1}^{\infty}-\frac{1}{\alpha_n}\cos\alpha_n\,(\xi + x)e^{-\alpha_n(b-2b_1+y)} = \frac{a}{2\pi}\ln\left(\operatorname{ch}t_3 - \cos x_1\right)$$

where:

$$t_0 = \frac{\pi}{a}(b+y); \qquad t_2 = \frac{\pi}{a}(b+2b_1-y); \qquad x_0 = \frac{\pi}{a}(\xi - x);$$
$$t_1 = \frac{\pi}{a}(b-y); \qquad t_3 = \frac{\pi}{a}(b-2b_1+y); \qquad x_1 = \frac{\pi}{a}(\xi + x);$$

Then the weakly convergent part for $W_2(x;y)$ will have the form:

$$\sum_{n=0}^{A} \tilde{a}(n) = \Upsilon \sum_{\kappa=1}^{4} [(G_{21}+1)D_\kappa + (G_{21}-1)D_{\kappa+4}]$$

here and further:

$$D_1 = -\ln(\operatorname{ch} t_0 - \cos x_0) \qquad D_5 = -\ln(\operatorname{ch} t_2 - \cos x_0)$$
$$D_2 = -\ln(\operatorname{ch} t_1 - \cos x_0) \qquad D_6 = -\ln(\operatorname{ch} t_3 - \cos x_0)$$
$$D_3 = \ln(\operatorname{ch} t_0 - \cos x_1) \qquad D_7 = \ln(\operatorname{ch} t_2 - \cos x_1)$$
$$D_4 = \ln(\operatorname{ch} t_1 - \cos x_1) \qquad D_8 = \ln(\operatorname{ch} t_3 - \cos x_1)$$
$$\Upsilon_2 = -\frac{a2G_{32}}{4\pi(R_3-1)}$$

Then $W_2(x;y)$ will take the form:

$$W_2(x;y) = \frac{2}{aG_3}\int_0^a p(\xi)\left[\Upsilon_2 \sum_{\kappa=1}^{4} [(G_{21}+1)D_\kappa + (G_{21}-1)D_{\kappa+4}] + \right.$$
$$\left. + \frac{1}{2}\sum_{n=1}^{A} \Xi(x,\xi)[Q_3 - Q_4]\right] d\xi \quad (2.15)$$

here and further

$$Q_3 = \frac{2G_{32}\left((G_{21}-1)\,T_2 + (G_{21}+1)\,T_1\right)}{\alpha_n\left(\left(-1+e^{-2\alpha_n b}\right)(R_3-1)+S\right)}$$
$$Q_4 = -\frac{2G_{32}\left((G_{21}-1)\,T_2 + (G_{21}+1)\,T_1\right)}{\alpha_n(R_3-1)}$$

2.4.4. Summary of weakly convergent parts of $\tau_{yz}^{2}(x;y)$

$$\frac{\partial W_2}{\partial y}(x;y) = \frac{2G_2}{aG_3}\int_0^a p(\xi)\left[\Upsilon_2\sum_{\kappa=1}^{4}\left[(G_{21}+1)D_{\kappa}^{'}+(G_{21}-1)D_{\kappa+4}^{'}\right]+\right.$$
$$\left.+\frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_3^{'}-Q_4^{'}]\right]d\xi \quad (2.16)$$

Where:

$$D_1^{'} = -\frac{\pi}{a}\frac{\operatorname{sh} t_0}{(\operatorname{ch} t_0-\cos x_0)} \qquad D_5^{'} = \frac{\pi}{a}\frac{\operatorname{sh} t_2}{(\operatorname{ch} t_2-\cos x_0)}$$
$$D_2^{'} = \frac{\pi}{a}\frac{\operatorname{sh} t_1}{(\operatorname{ch} t_1-\cos x_0)} \qquad D_6^{'} = -\frac{\pi}{a}\frac{\operatorname{sh} t_3}{(\operatorname{ch} t_3-\cos x_0)}$$
$$D_3^{'} = -\frac{\pi}{a}\frac{\operatorname{sh} t_0}{(\operatorname{ch} t_0-\cos x_1)} \qquad D_7^{'} = -\frac{\pi}{a}\frac{\operatorname{sh} t_2}{(\operatorname{ch} t_2-\cos x_1)}$$
$$D_4^{'} = \frac{\pi}{a}\frac{\operatorname{sh} t_1}{(\operatorname{ch} t_1-\cos x_1)} \qquad D_8^{'} = \frac{\pi}{a}\frac{\operatorname{sh} t_3}{(\operatorname{ch} t_3-\cos x_1)}$$
$$Q_3^{'} = \frac{2G_{32}\left((G_{21}-1)\,T_2+(G_{21}+1)\,T_1\right)}{\alpha_n\left(\left(-1+e^{-2\alpha_n b}\right)(R_3-1)+S\right)}$$
$$Q_4^{'} = -\frac{2G_{32}\left((G_{21}-1)\,T_2+(G_{21}+1)\,T_1\right)}{\alpha_n(R_3-1)}$$
$$T_1^{'} = -\alpha_n(-e^{-\alpha_{nn}(b-y)}+e^{-\alpha_{nn}(b+y)})$$
$$T_2^{'} = -\alpha_n(-e^{-\alpha_n(b+2b_1-y)}+e^{-\alpha_n(b-2b_1+y)})$$

2.4.5. Summary of weakly convergent parts of $W_3(x;y)$

$$W_3\left(x,y\right)=$$
$$\frac{2}{a}\int_0^a\sum_{n=0}^{\infty}\frac{p_{\alpha_n}\left(T_1\left(F_1+1\right)+T_2\left(F_2-1\right)+T_3\left(F_3-1\right)+T_4\left(R_5+1\right)\right)}{G_3\alpha_n\left(\left(-1+e^{-2\alpha_n b}\right)\left(R_3-1\right)+S\right)}\,\mathrm{d}\xi$$

where: $T_1=e^{-\alpha_n(b+y)}+e^{-\alpha_n(b-y)}$,
$T_2=e^{-\alpha_n(b+2b_1-y)}+e^{-\alpha_n(b-2b_1+y)}$,
$T_3=e^{-\alpha_n(b-2b_2+y)}+e^{-\alpha_n(b+2b_2-y)}$,
$T_4=e^{-\alpha_n(b+2b_1-2b_2+y)}+e^{-\alpha_n(b-2b_1+2b_2-y)}$,
$\Omega(x,\xi)=\frac{1}{2}(\cos\alpha_{nn}(\xi-x)-\cos\alpha_{nn}(\xi+x))$
Applying the formula (2.12) it is obtained:

$$\sum_{n=1}^{\infty}\frac{1}{\alpha_n}\cos\alpha_n\left(\xi-x\right)e^{-\alpha_n(b+y)}=-\frac{a}{2\pi}\ln\left(\operatorname{ch}t_0-\cos x_0\right)$$
$$\sum_{n=1}^{\infty}\frac{1}{\alpha_n}\cos\alpha_n\left(\xi-x\right)e^{-\alpha_n(b-y)}=-\frac{a}{2\pi}\ln\left(\operatorname{ch}t_1-\cos x_0\right)$$
$$\sum_{n=1}^{\infty}-\frac{1}{\alpha_n}\cos\alpha_n\left(\xi+x\right)e^{-\alpha_n(b+y)}=\frac{a}{2\pi}\ln\left(\operatorname{ch}t_0-\cos x_1\right)$$
$$\sum_{n=1}^{\infty}-\frac{1}{\alpha_n}\cos\alpha_n\left(\xi+x\right)e^{-\alpha_n(b-y)}=\frac{a}{2\pi}\ln\left(\operatorname{ch}t_1-\cos x_1\right)$$
$$\sum_{n=1}^{\infty}\frac{1}{\alpha_n}\cos\alpha_n\left(\xi-x\right)e^{-\alpha_n(b+2b_1-y)}=-\frac{a}{2\pi}\ln\left(\operatorname{ch}t_2-\cos x_0\right)$$
$$\sum_{n=1}^{\infty}\frac{1}{\alpha_n}\cos\alpha_n\left(\xi-x\right)e^{-\alpha_n(b-2b_1+y)}=-\frac{a}{2\pi}\ln\left(\operatorname{ch}t_3-\cos x_0\right)$$
$$\sum_{n=1}^{\infty}-\frac{1}{\alpha_n}\cos\alpha_n\left(\xi+x\right)e^{-\alpha_n(b+2b_1-y)}=\frac{a}{2\pi}\ln\left(\operatorname{ch}t_2-\cos x_1\right)$$
$$\sum_{n=1}^{\infty}-\frac{1}{\alpha_n}\cos\alpha_n\left(\xi+x\right)e^{-\alpha_n(b-2b_1+y)}=\frac{a}{2\pi}\ln\left(\operatorname{ch}t_3-\cos x_1\right)$$
$$\sum_{n=1}^{\infty}\frac{1}{\alpha_n}\cos\alpha_n\left(\xi-x\right)e^{-\alpha_n(b+2b_2-y)}=-\frac{a}{2\pi}\ln\left(\operatorname{ch}t_4-\cos x_0\right)$$

$$\sum_{n=1}^{\infty} \frac{1}{\alpha_n} \cos \alpha_n (\xi - x) e^{-\alpha_n (b - 2b_2 + y)} = -\frac{a}{2\pi} \ln(\operatorname{ch} t_5 - \cos x_0)$$
$$\sum_{n=1}^{\infty} -\frac{1}{\alpha_n} \cos \alpha_n (\xi + x) e^{-\alpha_n (b + 2b_2 - y)} = \frac{a}{2\pi} \ln(\operatorname{ch} t_4 - \cos x_1)$$
$$\sum_{n=1}^{\infty} -\frac{1}{\alpha_n} \cos \alpha_n (\xi + x) e^{-\alpha_n (b - 2b_2 + y)} = \frac{a}{2\pi} \ln(\operatorname{ch} t_5 - \cos x_1)$$
$$\sum_{n=1}^{\infty} \frac{1}{\alpha_n} \cos \alpha_n (\xi - x) e^{-\alpha_n (b + 2b_1 - 2b_2 + y)} = -\frac{a}{2\pi} \ln(\operatorname{ch} t_6 - \cos x_0)$$
$$\sum_{n=1}^{\infty} \frac{1}{\alpha_n} \cos \alpha_n (\xi - x) e^{-\alpha_n (b - 2b_1 + 2b_2 - y} = -\frac{a}{2\pi} \ln(\operatorname{ch} t_7 - \cos x_0)$$
$$\sum_{n=1}^{\infty} -\frac{1}{\alpha_n} \cos \alpha_n (\xi + x) e^{-\alpha_n (b + 2b_1 - 2b_2 + y)} = \frac{a}{2\pi} \ln(\operatorname{ch} t_6 - \cos x_1)$$
$$\sum_{n=1}^{\infty} -\frac{1}{\alpha_n} \cos \alpha_n (\xi + x) e^{-\alpha_n (b - 2b_1 + 2b_2 - y)} = \frac{a}{2\pi} \ln(\operatorname{ch} t_7 - \cos x_1)$$

where:

$$t_0 = \frac{\pi}{a}(b + y); \qquad t_5 = \frac{\pi}{a}(b - 2b_2 + y);$$
$$t_1 = \frac{\pi}{a}(b - y); \qquad t_6 = \frac{\pi}{a}(b + 2b_1 - 2b_2 + y);$$
$$t_2 = \frac{\pi}{a}(b + 2b_1 - y); \qquad t_7 = \frac{\pi}{a}(b - 2b_1 + 2b_2 - y);$$
$$t_3 = \frac{\pi}{a}(b - 2b_1 + y); \qquad x_0 = \frac{\pi}{a}(\xi - x);$$
$$t_4 = \frac{\pi}{a}(b + 2b_2 - y); \qquad x_1 = \frac{\pi}{a}(\xi + x);$$

Then the weakly convergent part for $W_3(x; y)$ will have the form:

$$\sum_{n=0}^{A} \tilde{a}(n) = \Upsilon_3 \sum_{\theta=1}^{4} [(F_1 + 1)H_\theta + (F_2 - 1)H_{\theta+4} + (F_3 - 1)H_{\theta+8} + (R_5 + 1)H_{\theta+12}]$$

here and further:

$$H_1 = -\ln(\operatorname{ch} t_0 - \cos x_0) \qquad H_6 = -\ln(\operatorname{ch} t_3 - \cos x_0)$$
$$H_2 = -\ln(\operatorname{ch} t_1 - \cos x_0) \qquad H_7 = \ln(\operatorname{ch} t_2 - \cos x_1)$$
$$H_3 = \ln(\operatorname{ch} t_0 - \cos x_1) \qquad H_8 = \ln(\operatorname{ch} t_3 - \cos x_1)$$
$$H_4 = \ln(\operatorname{ch} t_1 - \cos x_1) \qquad H_9 = -\ln(\operatorname{ch} t_4 - \cos x_0)$$
$$H_5 = -\ln(\operatorname{ch} t_2 - \cos x_0) \qquad H_{10} = -\ln(\operatorname{ch} t_5 - \cos x_0)$$

$$H_{11} = \ln(\operatorname{ch} t_4 - \cos x_1) \qquad H_{14} = -\ln(\operatorname{ch} t_7 - \cos x_0)$$
$$H_{12} = \ln(\operatorname{ch} t_5 - \cos x_1) \qquad H_{15} = \ln(\operatorname{ch} t_6 - \cos x_1)$$
$$H_{13} = -\ln(\operatorname{ch} t_6 - \cos x_0) \qquad H_{16} = \ln(\operatorname{ch} t_7 - \cos x_1)$$
$$\Upsilon_3 = -\frac{1}{4\pi(R_3 - 1)}$$

Then $W_3(x;y)$ will take the form:

$$W_3(x;y) = \frac{2}{aG_3}\int_0^a p(\xi)\left[\Upsilon_3 \sum_{\theta=1}^{4}\{(F_1+1)H_\theta + (F_2-1)H_{\theta+4} + (F_3-1)H_{\theta+8} + (R_5+1)H_{\theta+12}\} + \frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_5 - Q_6]\right]d\xi \quad (2.17)$$

Here and further:

$$Q_5 = \frac{T_1(F_1+1) + T_2(F_2-1) + T_3(F_3-1) + T_4(R_5+1)}{G_3\alpha_n\left(\left(-1+e^{-2\alpha_n b}\right)(R_3-1) + S\right)}$$
$$Q_6 = -\frac{T_1(F_1+1) + T_2(F_2-1) + T_3(F_3-1) + T_4(R_5+1)}{G_3\alpha_n(R_3-1)}$$

2.4.6. Summary of weakly convergent parts of $\tau_{yz}^{3}(x;y)$

$$\frac{\partial W_3}{\partial y}(x;y)=\frac{2G_3}{aG_3}\int_0^a p(\xi)\left[\Upsilon_3\sum_{\theta=1}^{4}\Big\{(F_1+1)H_\theta^{'}+(F_2-1)H_{\theta+4}^{'}+(F_3-1)H_{\theta+8}^{'}+\right.$$

$$\left.+(R_5+1)H_{\theta+12}^{'}\Big\}+\frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_5^{'}-Q_6^{'}]\right]d\xi \quad (2.18)$$

here and further:

$$H_1^{'}=-\frac{\pi}{a}\frac{\operatorname{sh}t_0}{(\operatorname{ch}t_0-\cos x_0)} \qquad H_9^{'}=\frac{\pi}{a}\frac{\operatorname{sh}t_4}{(\operatorname{ch}t_4-\cos x_0)}$$

$$H_2^{'}=\frac{\pi}{a}\frac{\operatorname{sh}t_1}{(\operatorname{ch}t_1-\cos x_0)} \qquad H_{10}^{'}=-\frac{\pi}{a}\frac{\operatorname{sh}t_5}{(\operatorname{ch}t_5-\cos x_0)}$$

$$H_3^{'}=-\frac{\pi}{a}\frac{\operatorname{sh}t_0}{(\operatorname{ch}t_0-\cos x_1)} \qquad H_{11}^{'}=-\frac{\pi}{a}\frac{\operatorname{sh}t_4}{(\operatorname{ch}t_4-\cos x_1)}$$

$$H_4^{'}=\frac{\pi}{a}\frac{\operatorname{sh}t_1}{(\operatorname{ch}t_1-\cos x_1)} \qquad H_{12}^{'}=\frac{\pi}{a}\frac{\operatorname{sh}t_5}{(\operatorname{ch}t_5-\cos x_1)}$$

$$H_5^{'}=\frac{\pi}{a}\frac{\operatorname{sh}t_2}{(\operatorname{ch}t_2-\cos x_0)} \qquad H_{13}^{'}=-\frac{\pi}{a}\frac{\operatorname{sh}t_6}{(\operatorname{ch}t_6-\cos x_0)}$$

$$H_6^{'}=-\frac{\pi}{a}\frac{\operatorname{sh}t_3}{(\operatorname{ch}t_3-\cos x_0)} \qquad H_{14}^{'}=\frac{\pi}{a}\frac{\operatorname{sh}t_7}{(\operatorname{ch}t_7-\cos x_0)}$$

$$H_7^{'}=-\frac{\pi}{a}\frac{\operatorname{sh}t_2}{(\operatorname{ch}t_2-\cos x_1)} \qquad H_{15}^{'}=\frac{\pi}{a}\frac{\operatorname{sh}t_6}{(\operatorname{ch}t_6-\cos x_1)}$$

$$H_8^{'}=\frac{\pi}{a}\frac{\operatorname{sh}t_3}{(\operatorname{ch}t_3-\cos x_1)} \qquad H_{16}^{'}=-\frac{\pi}{a}\frac{\operatorname{sh}t_7}{(\operatorname{ch}t_7-\cos x_1)}$$

$$Q_5^{'}=\frac{T_1^{'}(F_1+1)+T_2^{'}(F_2-1)+T_3^{'}(F_3-1)+T_4^{'}(R_5+1)}{G_3\alpha_n\left(\left(-1+e^{-2\alpha_n b}\right)(R_3-1)+S\right)}$$

$$Q_6^{'}=-\frac{T_1^{'}(F_1+1)+T_2^{'}(F_2-1)+T_3^{'}(F_3-1)+T_4^{'}(R_5+1)}{G_3\alpha_n(R_3-1)}$$

$$T_1^{'}=-\alpha_n(-e^{-\alpha_{nn}(b-y)}+e^{-\alpha_{nn}(b+y)})$$
$$T_2^{'}=-\alpha_n(-e^{-\alpha_n(b+2b_1-y)}+e^{-\alpha_n(b-2b_1+y)})$$
$$T_3^{'}=-\alpha_n(-e^{-\alpha_n(b+2b_2-y)}+e^{-\alpha_n(b-2b_2+y)})$$
$$T_4^{'}=-\alpha_n(e^{-\alpha_n(b+2b_1-2b_2+y)}-e^{-\alpha_n(b-2b_1+2b_2-y)})$$

2.5. Analysis of numerical calculations

Calculations were made with the following parameters:
Area parameters:

- $a = 12$
- $b = 12$
- $b_1 = 3$
- $b_2 = 9$
- $G_1 = 8.0 \cdot 10^{10}$ — carbon steel
- $G_2 = 4.0 \cdot 10^{10}$ — manganese bronze
- $G_3 = 2.7 \cdot 10^{10}$ — rolled duralumin

Load parameters:

- $p_1(\xi) = \sin \frac{\pi}{a}\xi$
- $p_2(\xi) = \frac{2\xi}{a} - \frac{4\xi(\xi - \frac{a}{2})}{a^2}$

2.5.1. Dynamics of displacement changes

Consider the dynamics of changes in displacements for a three-layer rectangular area at different values of y.
For the first layer:

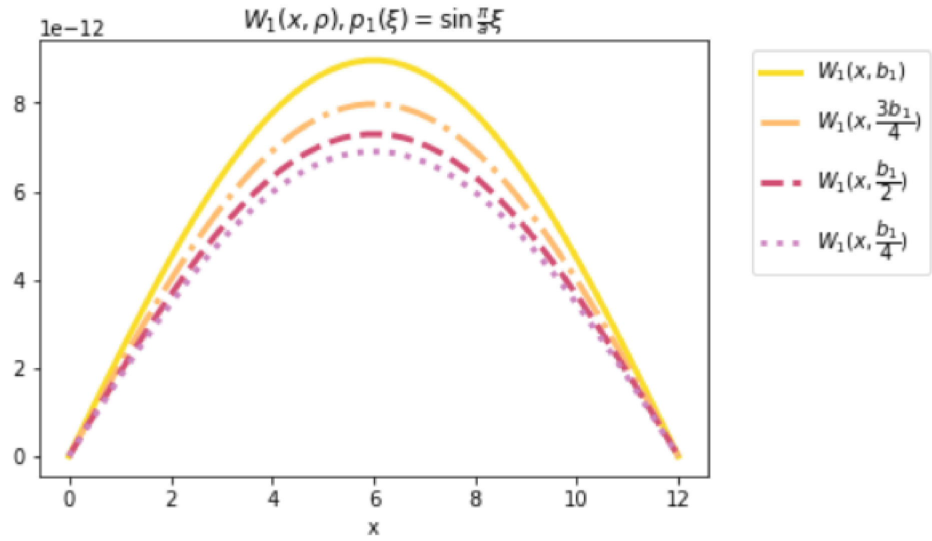


Figure 2.1. Displacement $W_1\,(x,y)$ at different values of y, 0<x<a

As can be seen from the figures (Fig.2.1), as the value of y approaches b_1, the displacement increases.
For the second layer::

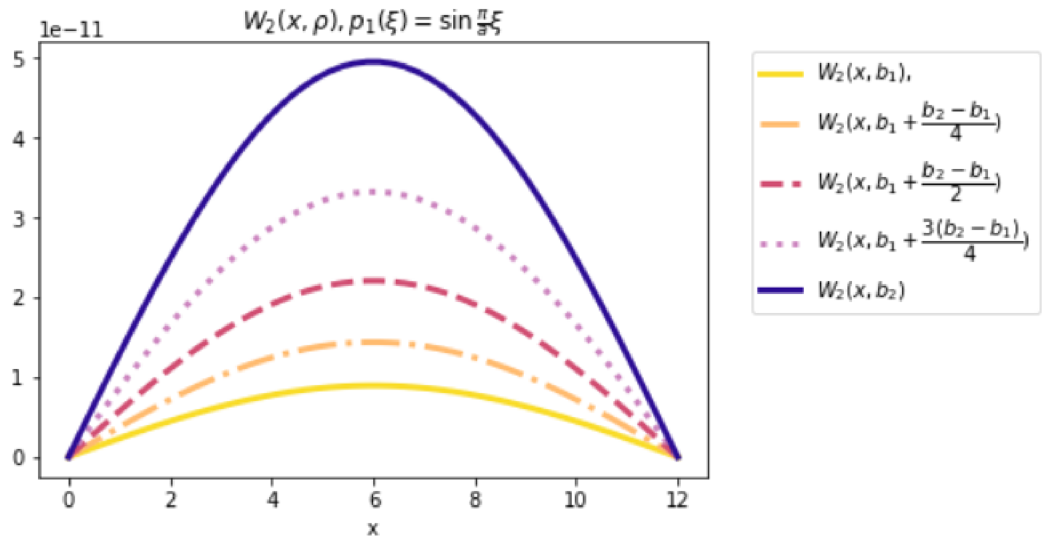


Figure 2.2. Displacement $W_2\,(x,y)$ at different values of y, 0<x<a

As can be seen from the figures (Fig.2.2), as the value of y approaches b_2, the displacement increases.

For the third layer::

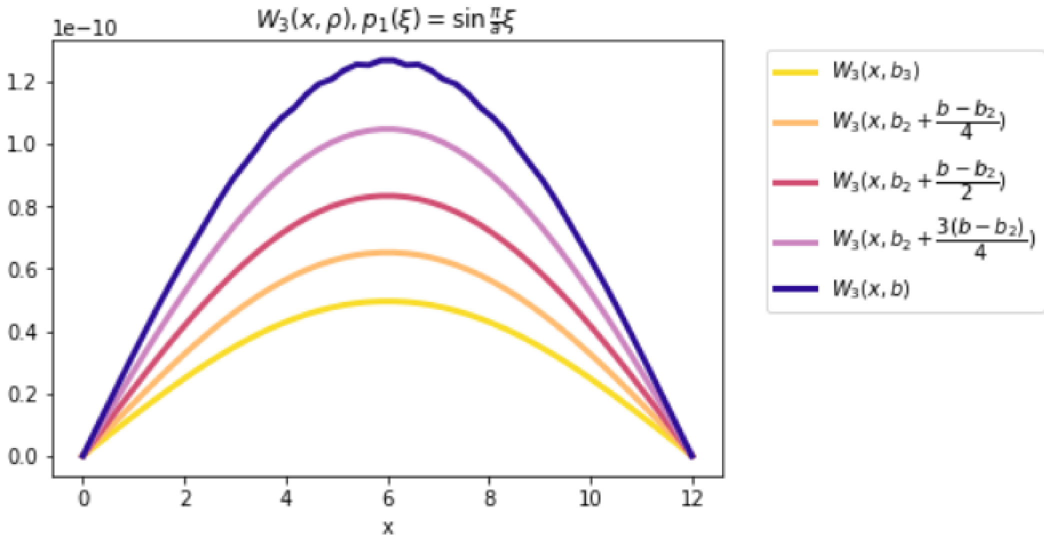


Figure 2.3. Displacement $W_3\,(x{,}y)$ at different values of y, 0<x<a

As can be seen from the figures (Fig.2.3), as the value of y approaches b, the displacement increases.

A displacement surface was also build with the same parameters.

For the first layer:

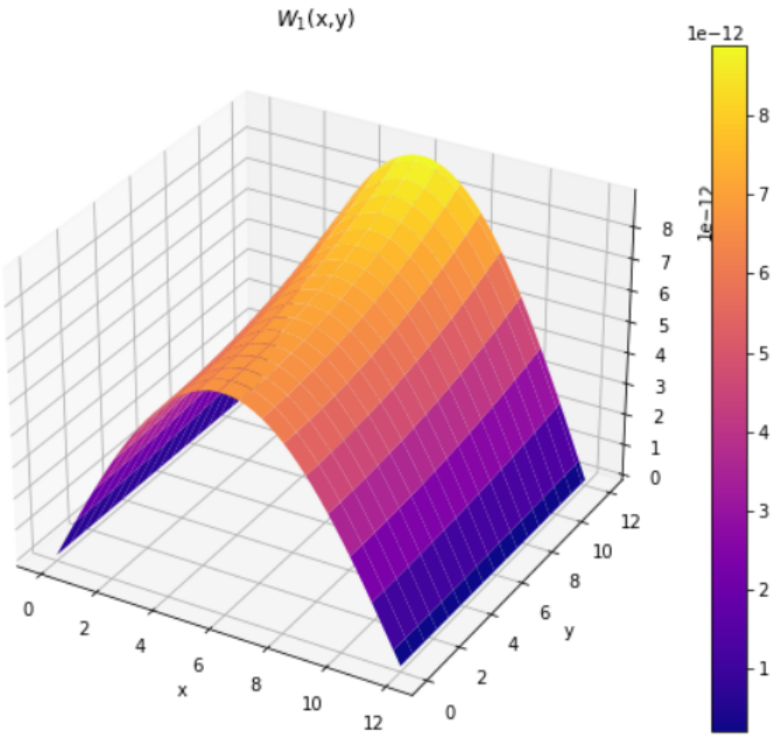


Figure 2.4. Surface $W_1\,(x{,}y)\,{,}p_1\,(\xi) = \sin\frac{\pi}{a}\xi$

For the second layer:

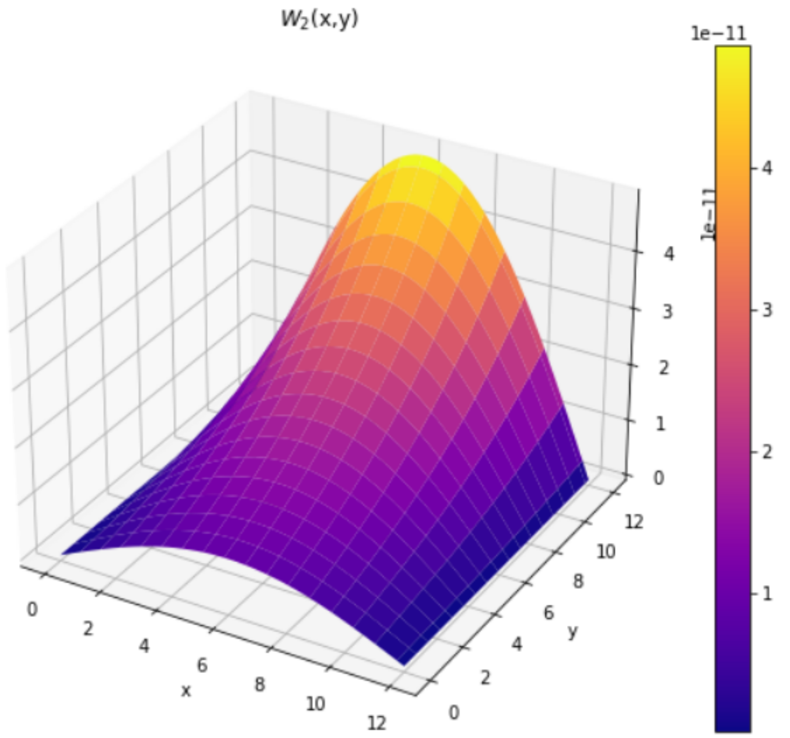


Figure 2.5. Surface $W_2\left(x,y\right), p_1\left(\xi\right) = \sin\frac{\pi}{a}\xi$

For the third layer:

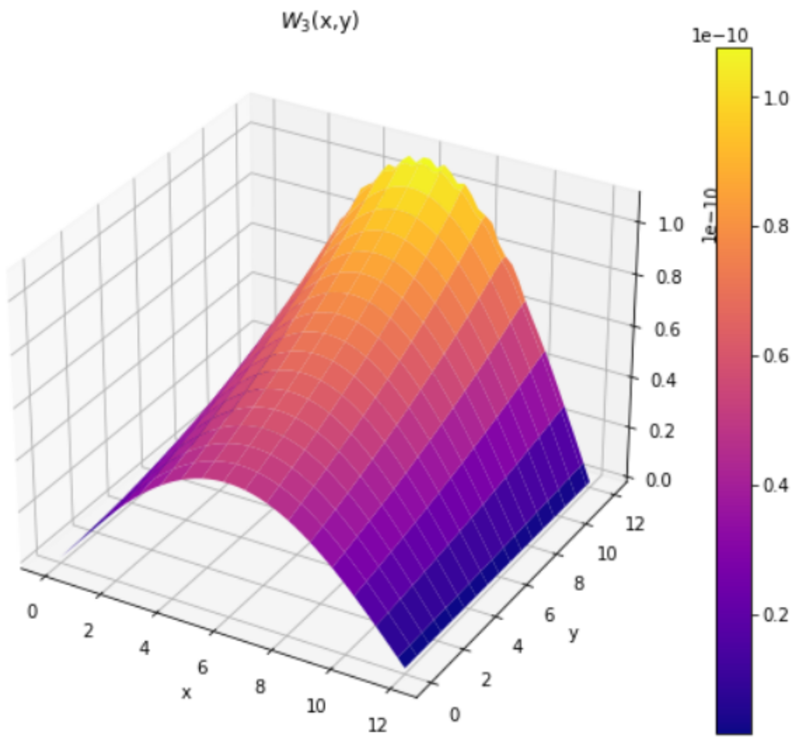


Figure 2.6. Surface $W_3\left(x,y\right), p_1\left(\xi\right) = \sin\frac{\pi}{a}\xi$

For all rectangular area:

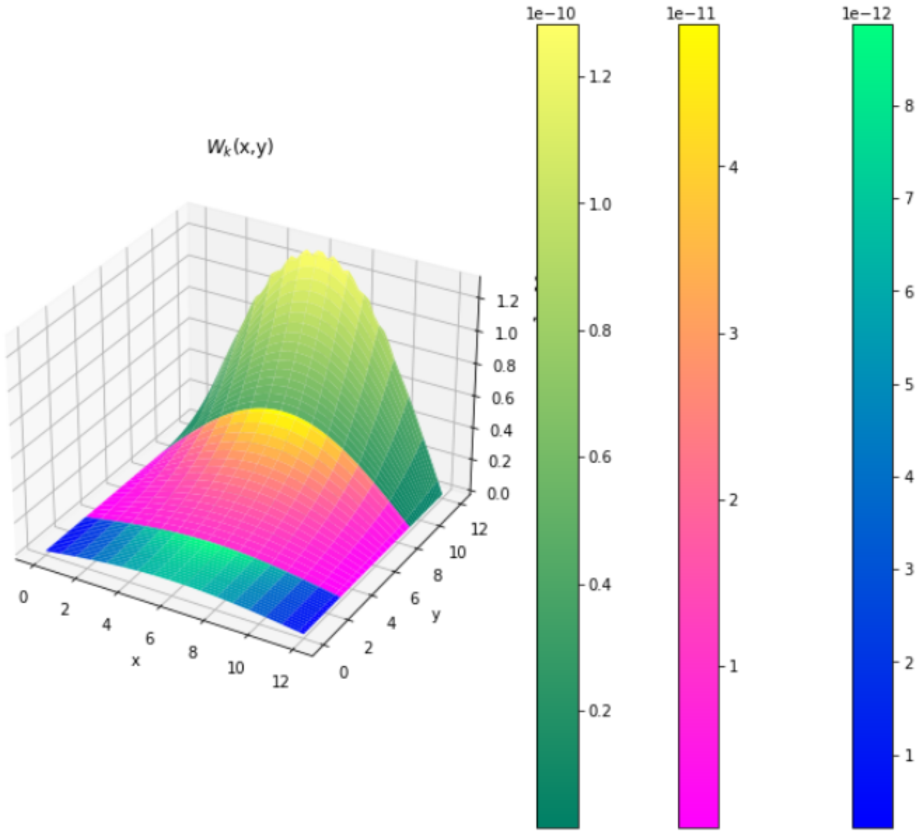


Figure 2.7. Surfaces $W_k\left(x,y\right), p_1\left(\xi\right) = \sin\frac{\pi}{a}\xi, k = 1,2,3$

2.5.2. Dynamics of stress changes

Consider the dynamics of stress changes for different layers of a rectangular area at different values of y.
For the first layer:

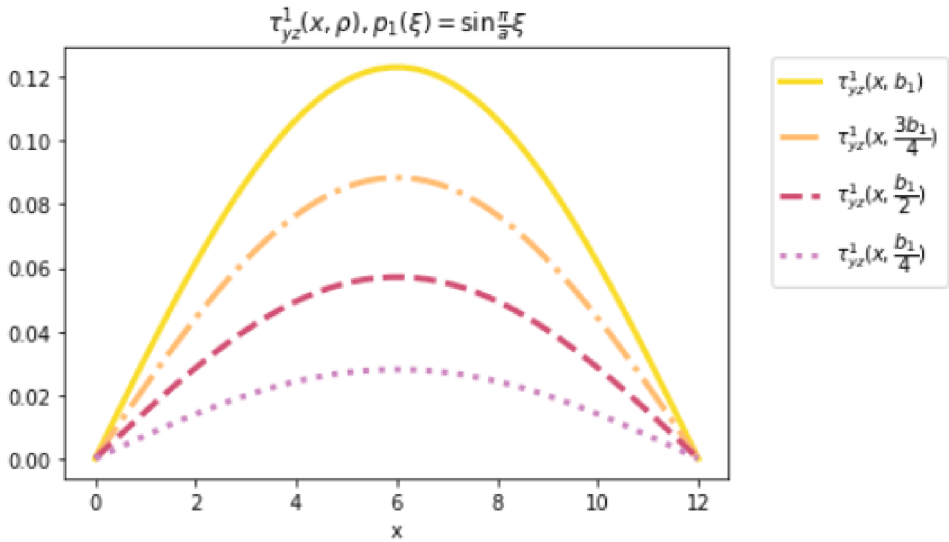


Figure 2.8. Stress $\tau^1_{yz}(x,y)$ at different values of $y, 0 < x < a$

It should be noted that the tendency of y to approach b_1 is the same as for displacement.
For the second layer:

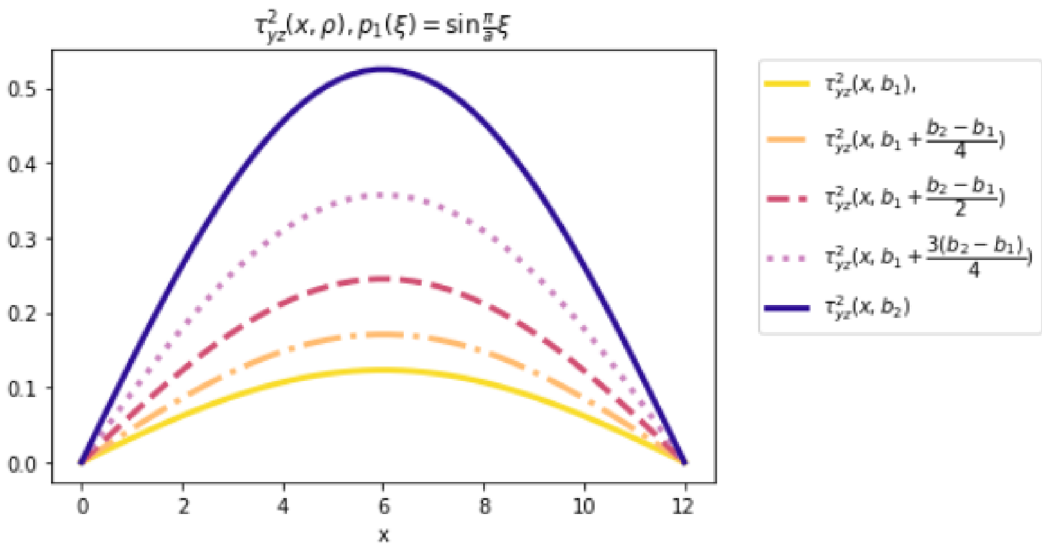


Figure 2.9. Stress $\tau^2_{yz}(x,y)$ at different values of $y, 0 < x < a$

It should be noted that the tendency of y to approach b_2 is the same as for

displacement.
For the third layer:

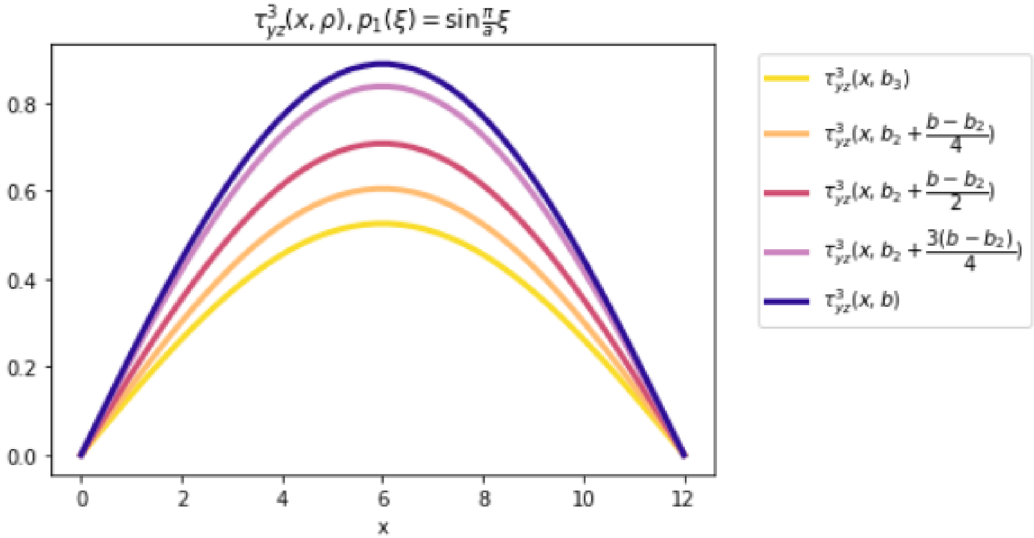


Figure 2.10. Stress $\tau^3_{yz}(x,y)$ at different values of $y, 0 < x < a$

It should be noted that the tendency of y to approach b is the same as for displacement.
A stressed surface was also constructed with the same parameters.
For the first layer:

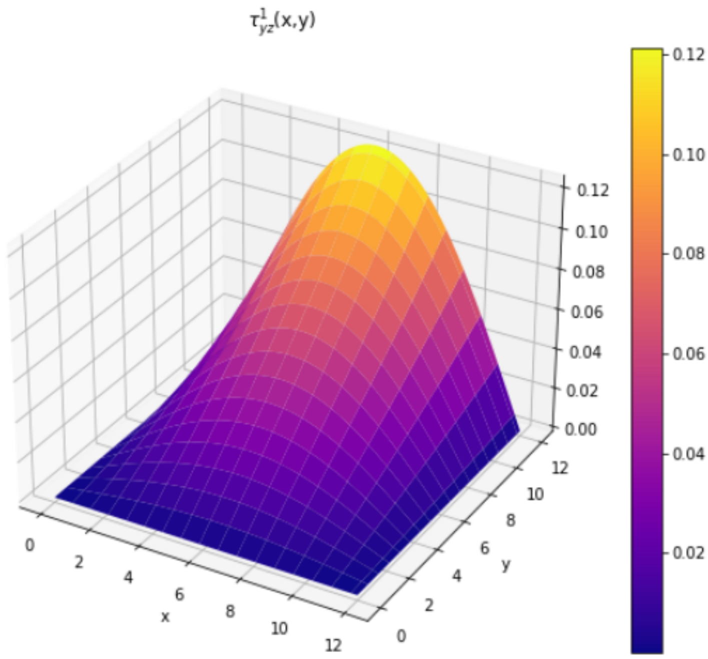


Figure 2.11. Surface $\tau^1_{yz}(x,y), p_1(\xi) = \sin\frac{\pi}{a}\xi$

For the second layer:

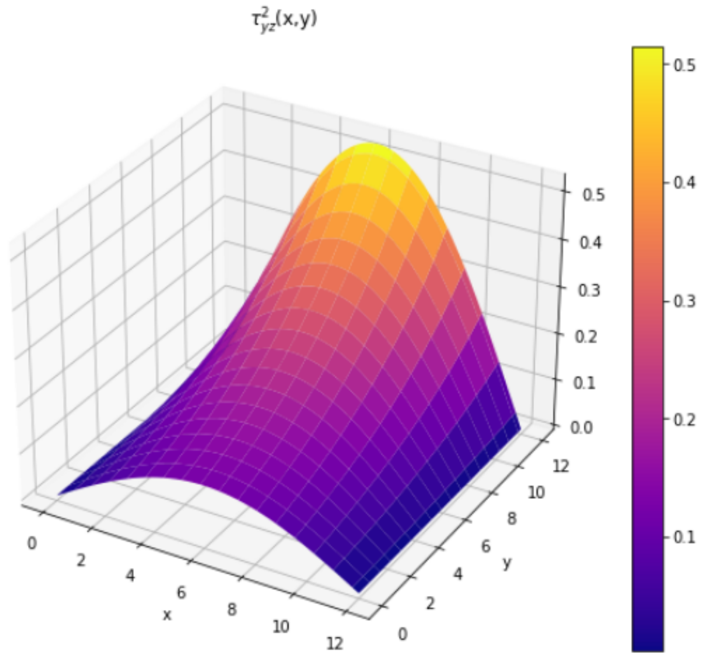


Figure 2.12. Surface $\tau^{2}_{yz}\,(x{,}y)\,{,}p_1\,(\xi) = \sin\frac{\pi}{a}\xi$

For the third layer:

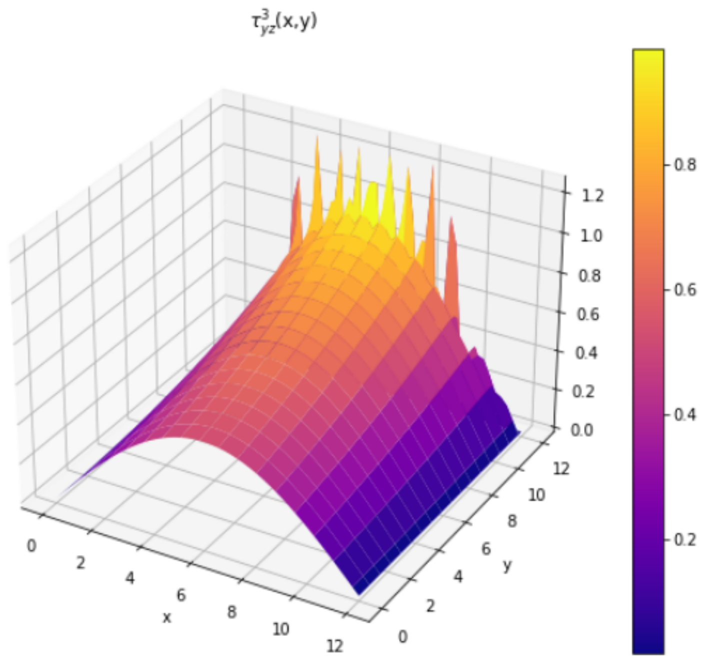


Figure 2.13. Surface $\tau^{3}_{yz}\,(x{,}y)\,{,}p_1\,(\xi) = \sin\frac{\pi}{a}\xi$

For the entire region:

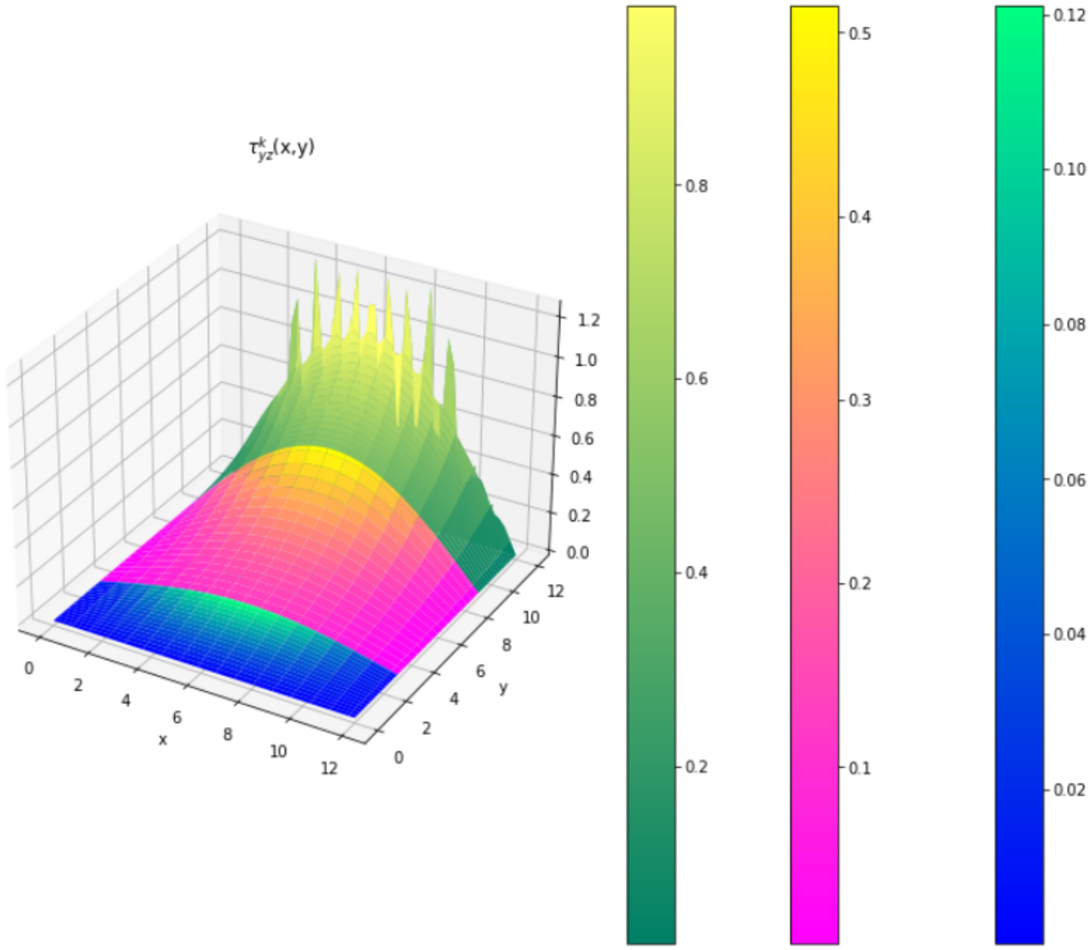
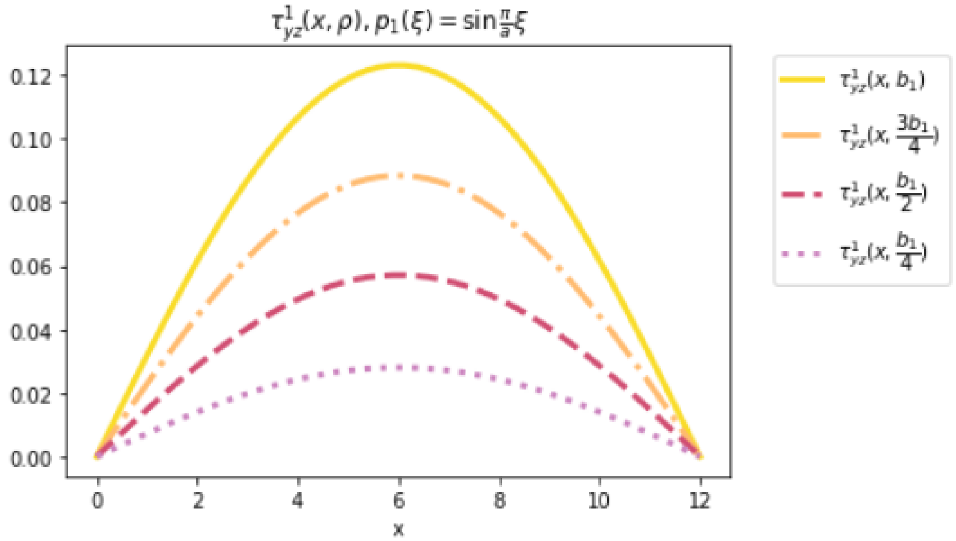


Figure 2.14. Surfaces $\tau^k_{yz}\,(x,y)\,,p_1\,(\xi) = \sin\frac{\pi}{a}\xi,k$ =1,2,3

2.5.3. Dependence of stress on the type of loads

Consider the dynamics of stress changes for different layers of a rectangular area under different loads $p(\xi)$ and at different values of y. Display the graphs together for a clearer analysis:
For the first layer:



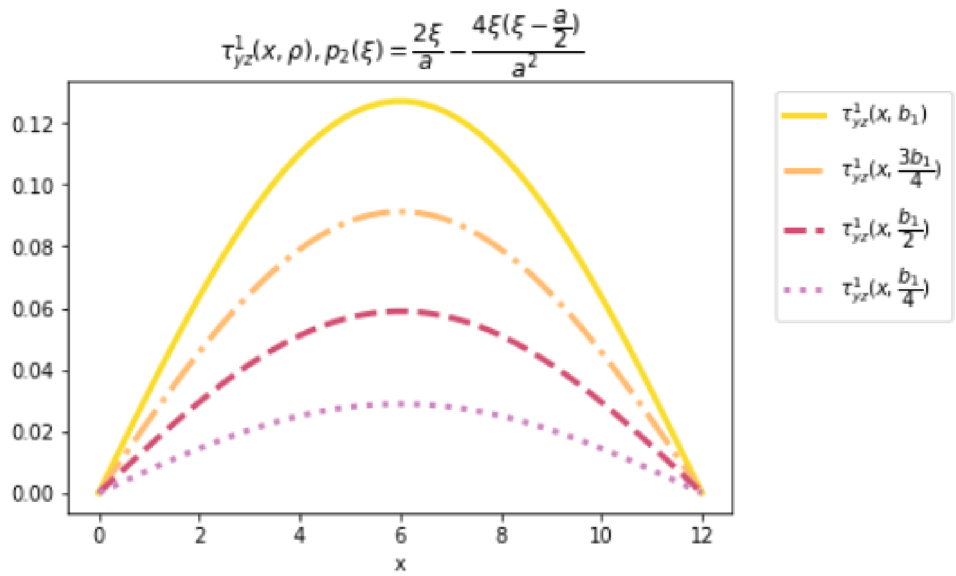
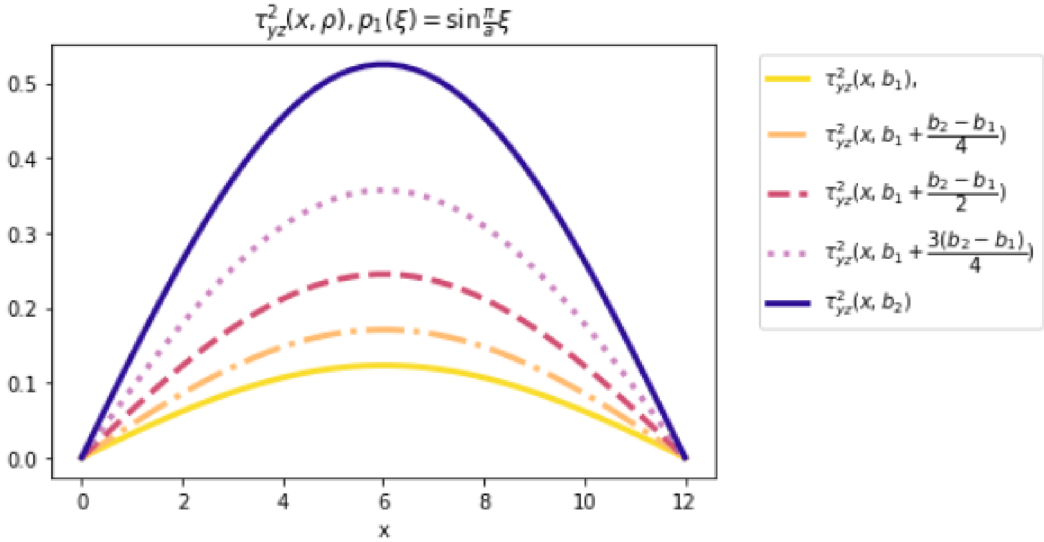


Table 2.1. The stress $\tau^1_{yz}\,(x,y)$ at different values of $y, 0 < x < a$ and types of loads $p(\xi)$

Note that the tendency of y to approach b_1 on both graphs coincides.

For the second layer:



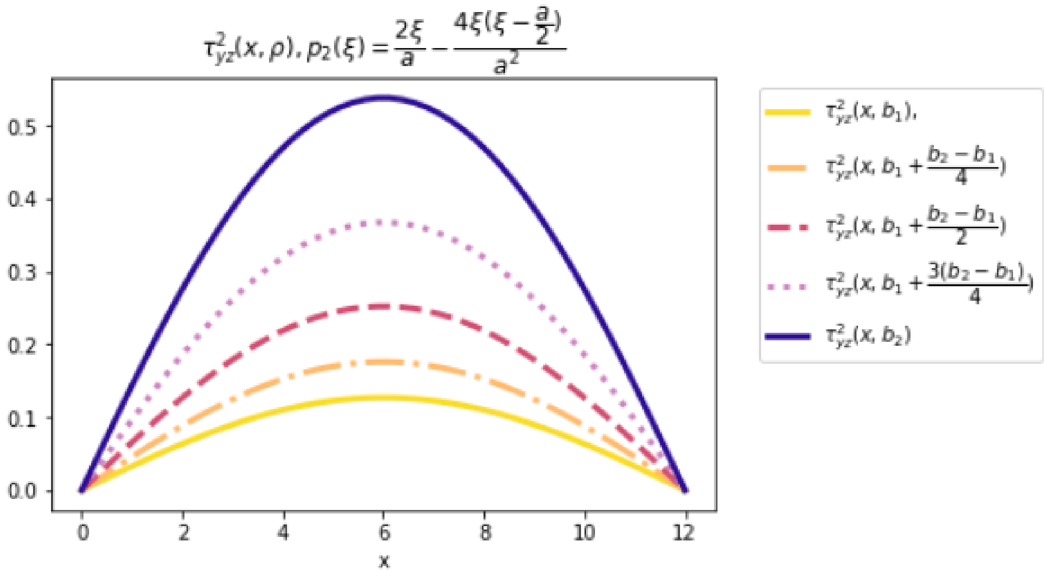
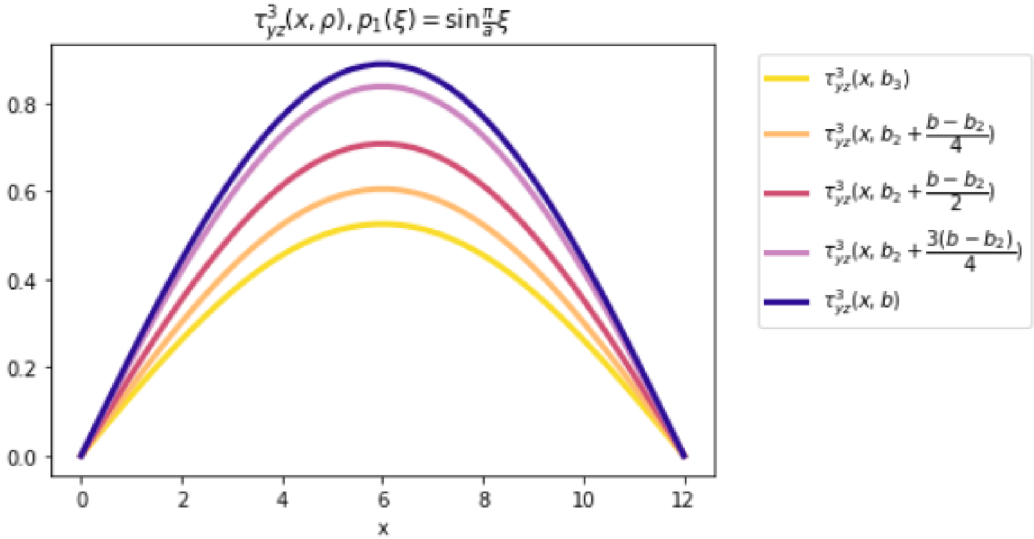


Table 2.2. The stress $\tau_{yz}^2\,(x,y)$ at different values of $y, 0 < x < a$ and types of loads $p(\xi)$

Note that the tendency of y to approach b_2 on both graphs coincides.

For the third layer:



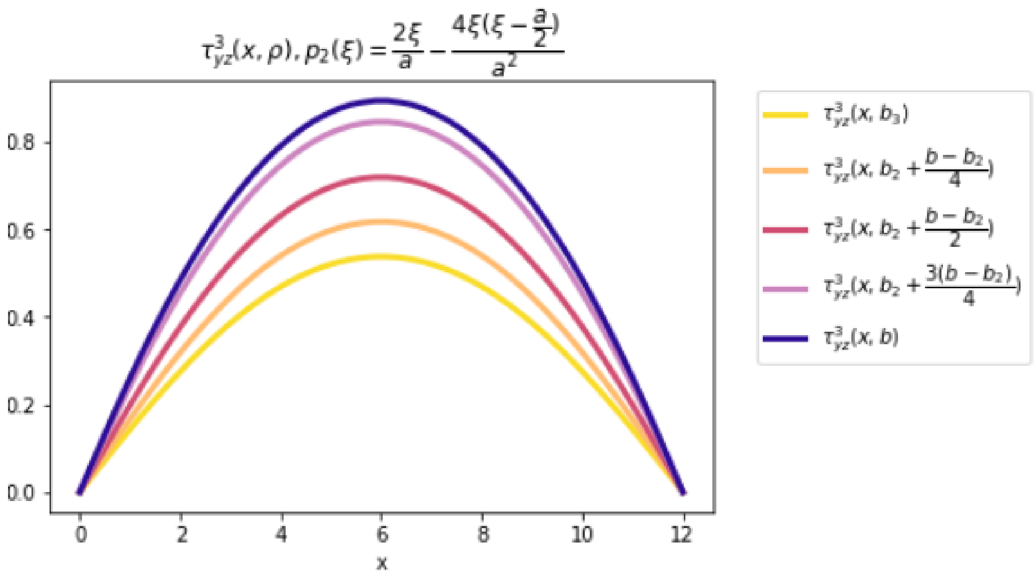
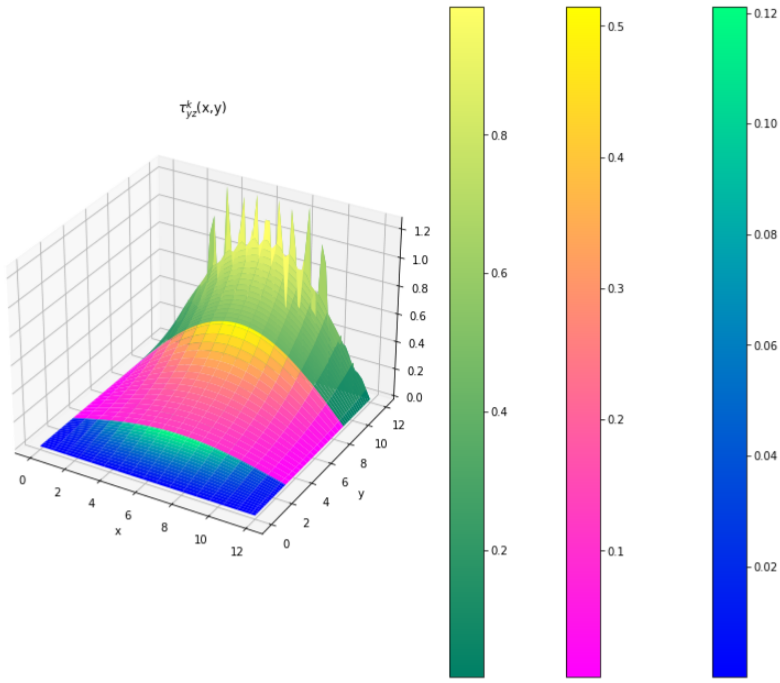


Table 2.3. The stress $\tau^3_{yz}(x,y)$ at different values of $y, 0 < x < a$ and types of loads $p(\xi)$

Note that the tendency of y to approach b on both graphs coincides.

Also, construct the stress surface of all layers with the same parameters.



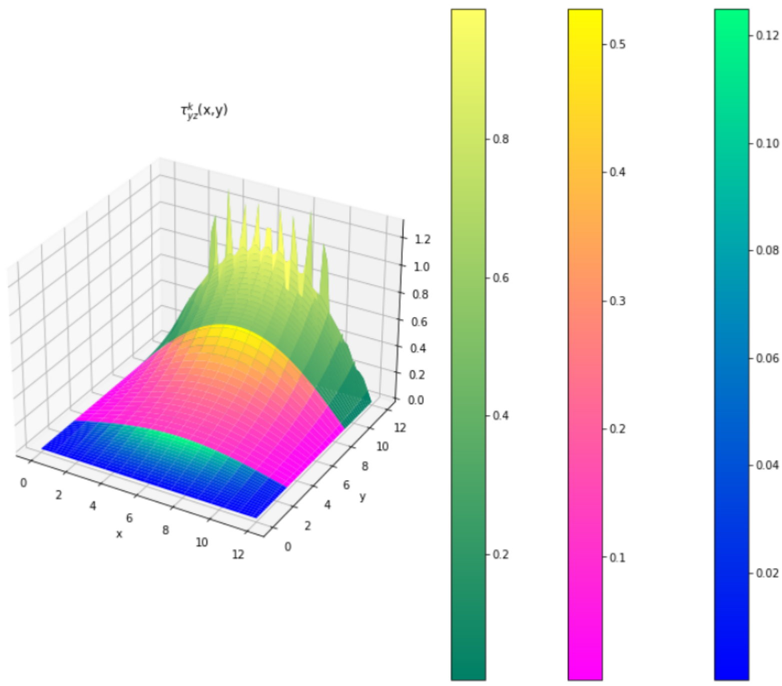


Table 2.4. The stress $\tau_{yz}^{k}\,(x,y)$ at different values of $y, 0 < x < a$ and types of loads $p(\xi)$,k=1,2 ,3

Analyzed all the graphs in this section, the conclusion is that the stress at the top of the area does not change depending on the type of load if we fulfil the condition $p(0) = 0$.

2.5.4. Dependence of stress on the geometry of a rectangular region

Consider the dependence of stress on the geometry of a two-layer rectangular region.
For the first layer:

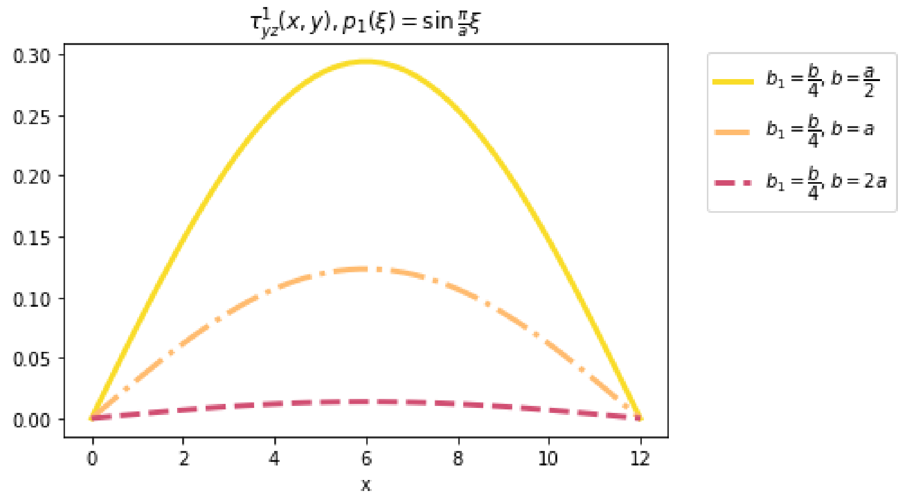


Figure 2.15. Investigation of the geometry of the region G_1

A more advanced dependency:

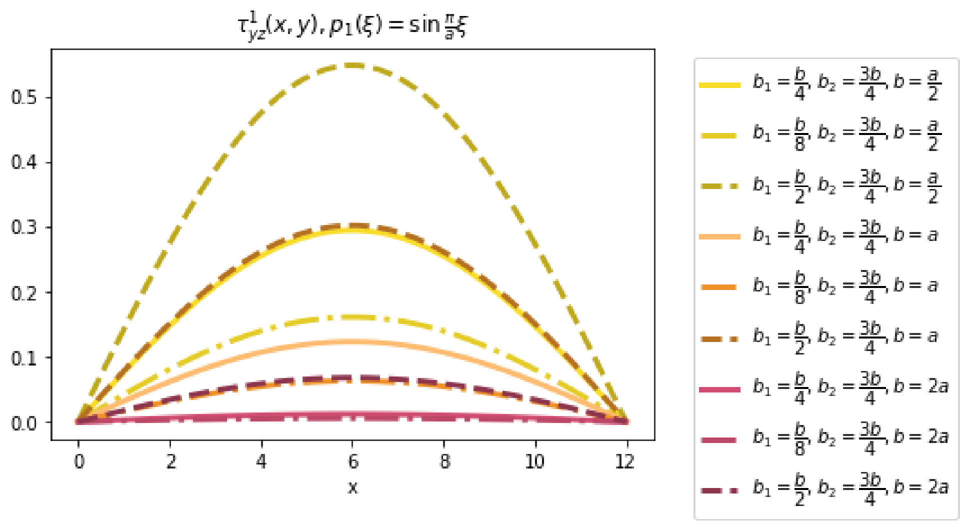


Figure 2.16. Investigation of the geometry of the region G_1

As can be seen from the graphs (Fig.2.15)"— (Fig.2.16) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.
It is also interesting to consider the dependence of stress on layer thicknesses:

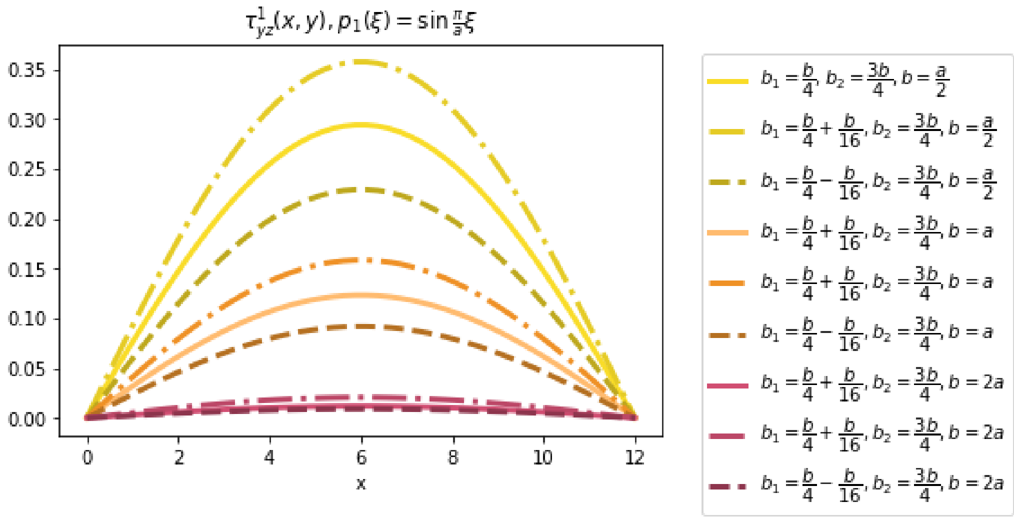


Figure 2.17. Investigation of the geometry of the region G_1

For the second layer:

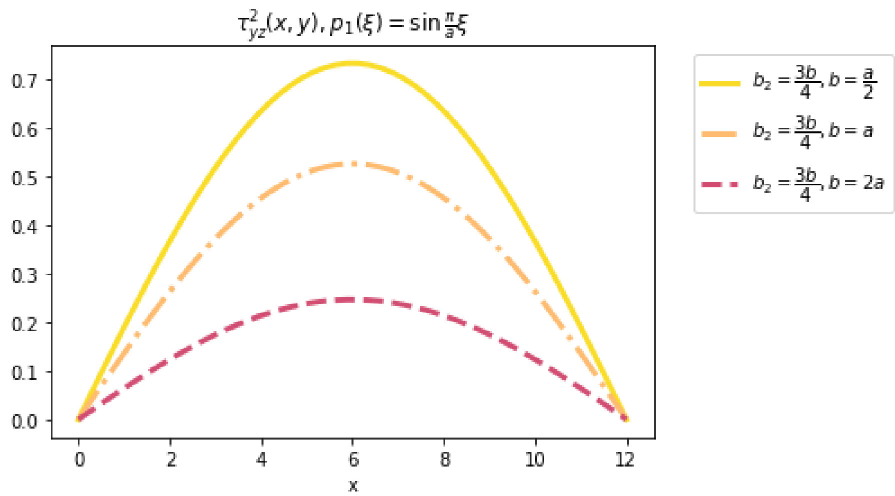


Figure 2.18. Investigation of the geometry of the region G_2

A more advanced dependency:

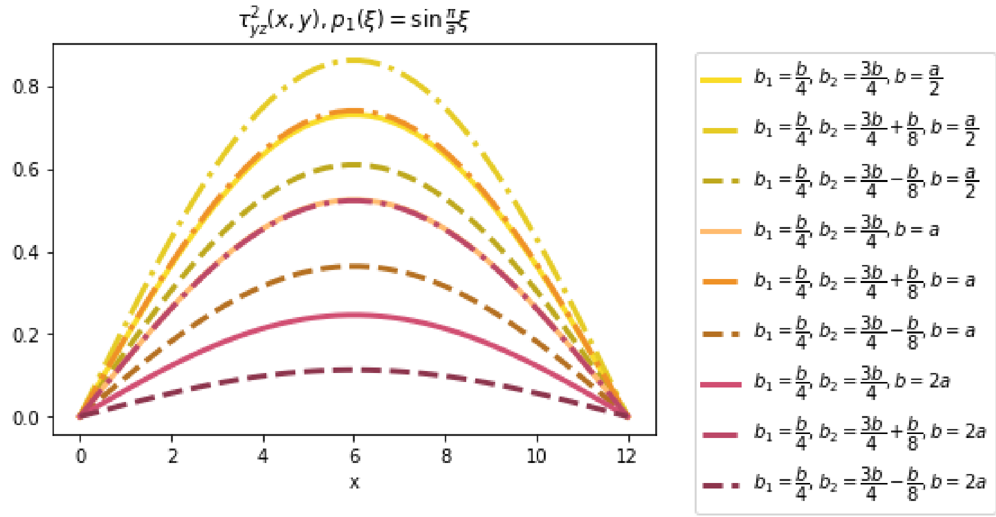


Figure 2.19. Investigation of the geometry of the region G_2

As can be seen from the graphs (Fig.2.18)"— (Fig.2.19) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.
It is also interesting to consider the dependence of stress on layer thicknesses:

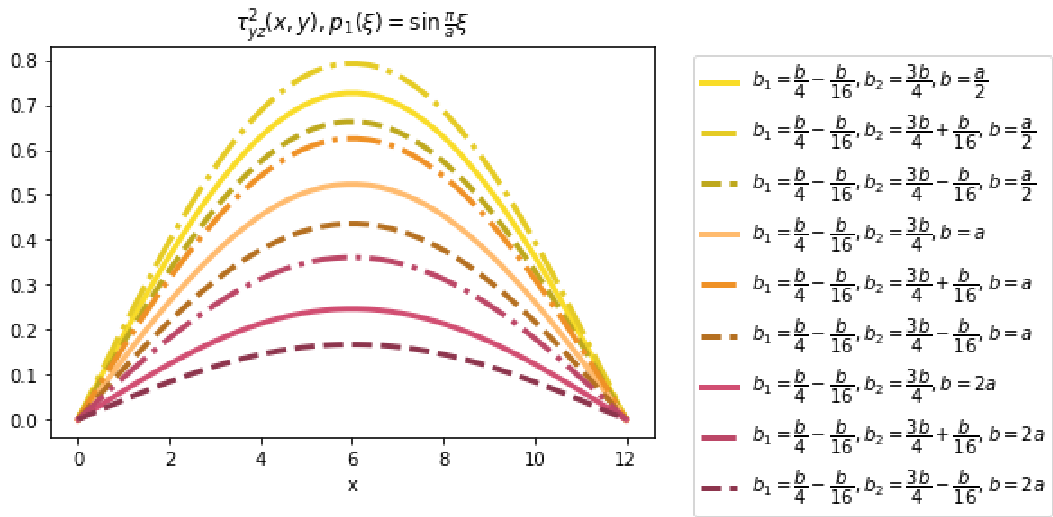


Figure 2.20. Investigation of the geometry of the region G_2

For the third layer:

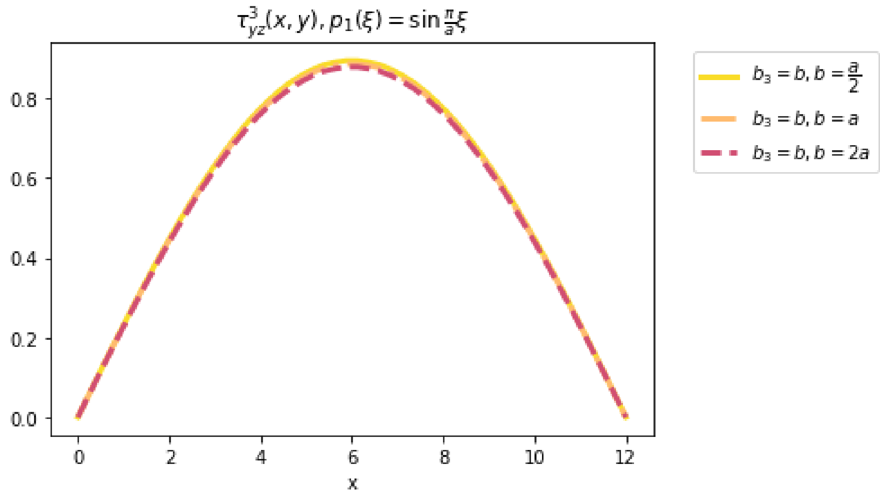


Figure 2.21. Investigation of the geometry of the region G_3

A more advanced dependency:

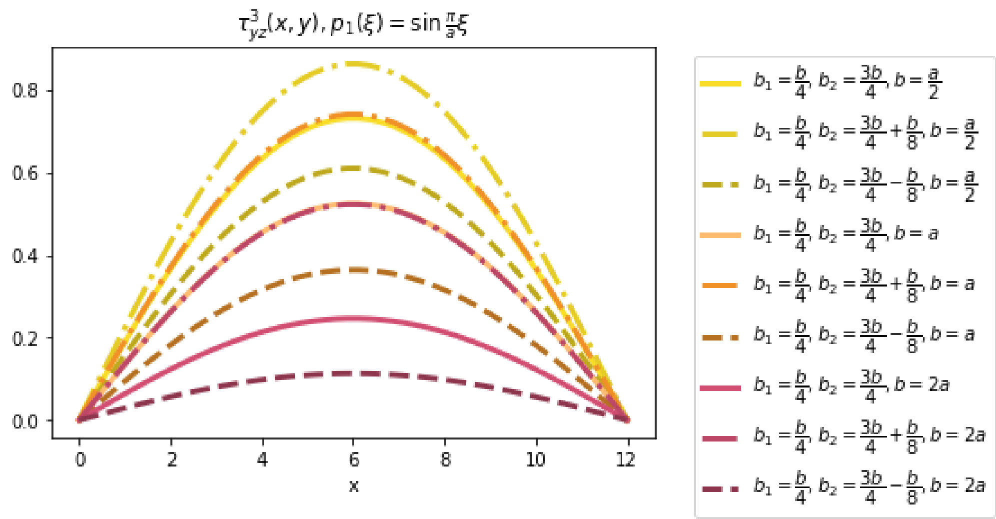


Figure 2.22. Investigation of the geometry of the region G_3

As can be seen from the graphs (Fig.2.21)"— (Fig.2.22) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.

It is also interesting to consider the dependence of stress on layer thicknesses:

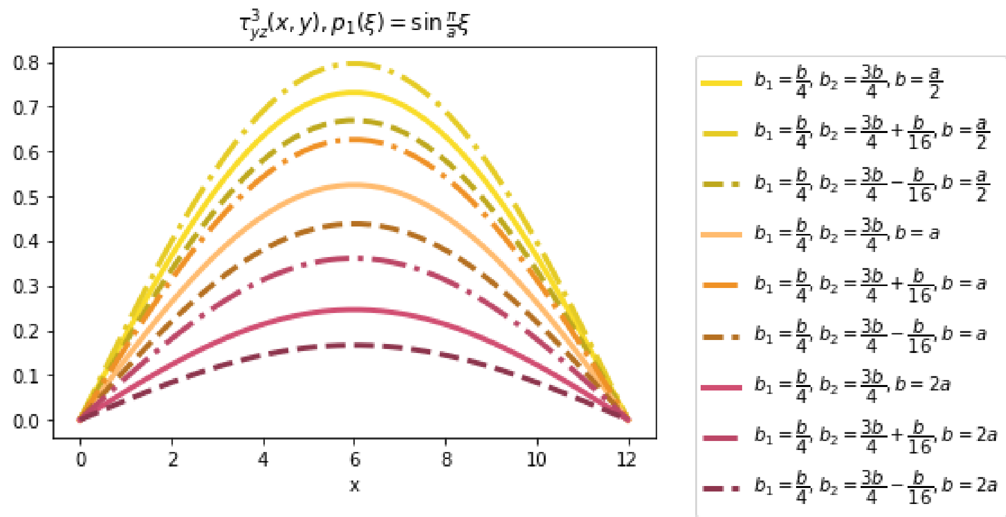


Figure 2.23. Investigation of the geometry of the region G_3

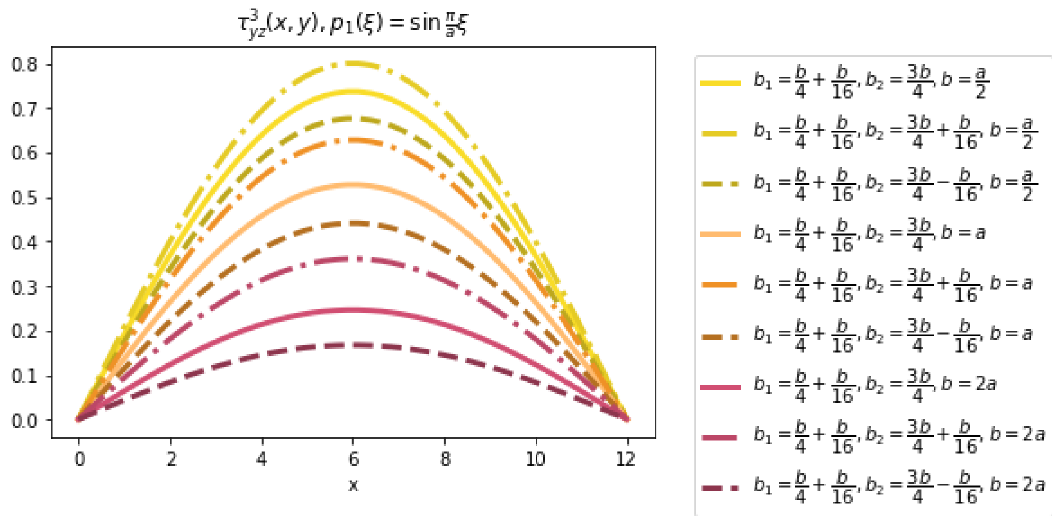


Figure 2.24. Investigation of the geometry of the region G_3

2.5.5. Dependence of the stress value on the arrangement of materials G_{132}

It is also necessary to consider the issue of the dependence of stress values on the material on which a load of intensity $p(x)$ acts Calculations were made with the following parameters:

Area parameters:

- $a = 12$
- $b = 12$
- $b_1 = 3$

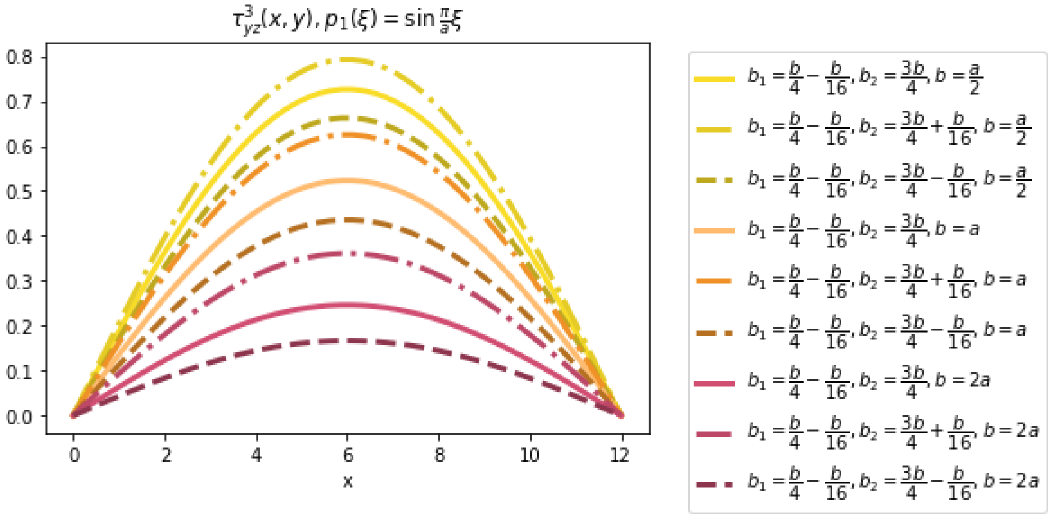


Figure 2.25. Investigation of the geometry of the region G_3

- $b_2 = 9$
- $G_1 = 8.0 \cdot 10^{10}$ "— carbon steel
- $G_2 = 2.7 \cdot 10^{10}$ "— duralumin
- $G_3 = 4.0 \cdot 10^{10}$ "— rolled manganese bronze

Load parameters:

- $p_1(\xi) = \sin\dfrac{\pi}{a}\xi$

Consider the dependence of stress on the geometry of a two-layer rectangular region.
For the first layer:

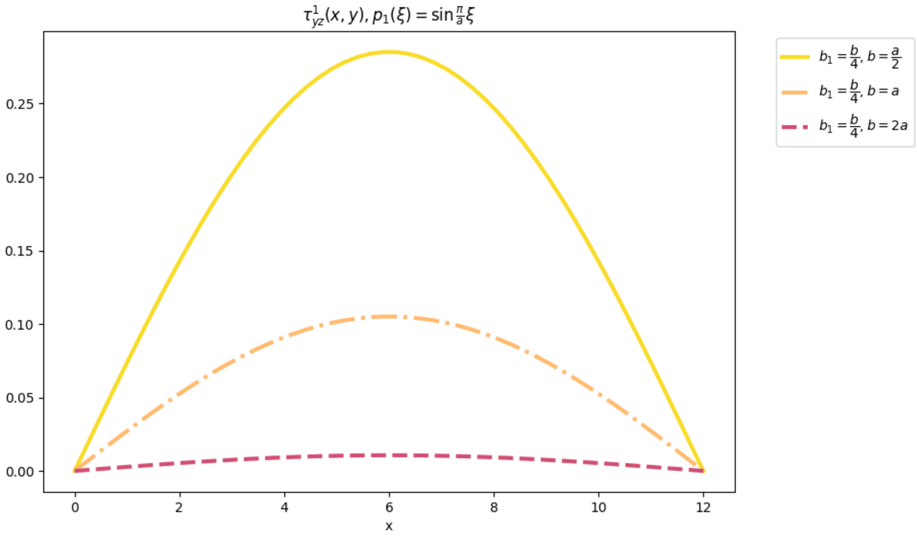


Figure 2.26. Investigation of the geometry of the region G_1

A more advanced dependency:

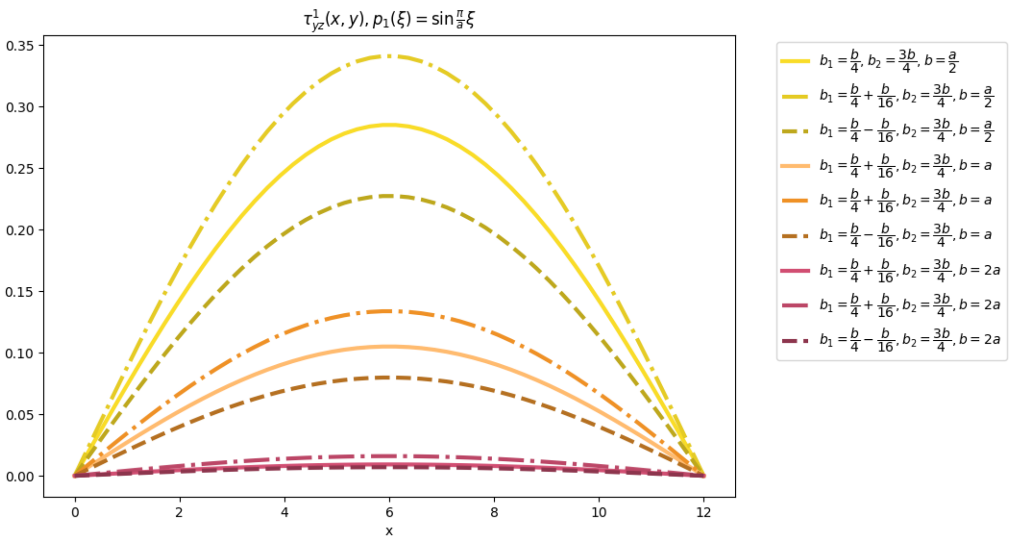


Figure 2.27. Investigation of the geometry of the region G_1

As can be seen from the graphs (Fig.2.26)"— (Fig.2.27) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.
It is also interesting to consider the dependence of stress on layer thicknesses:

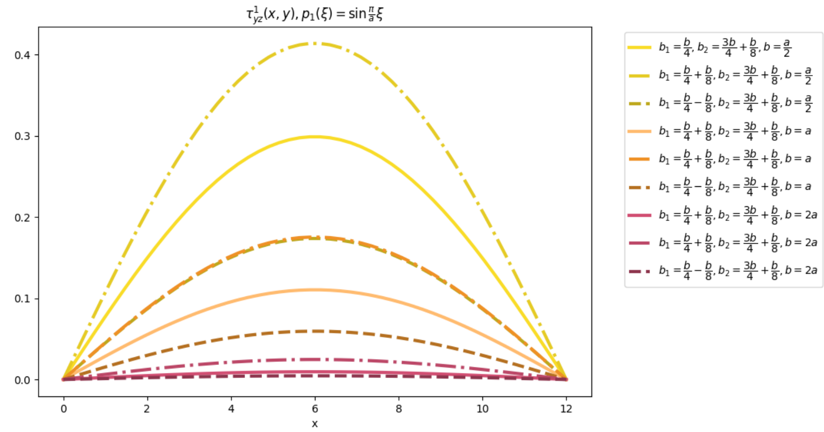


Figure 2.28. Investigation of the geometry of the region G_1

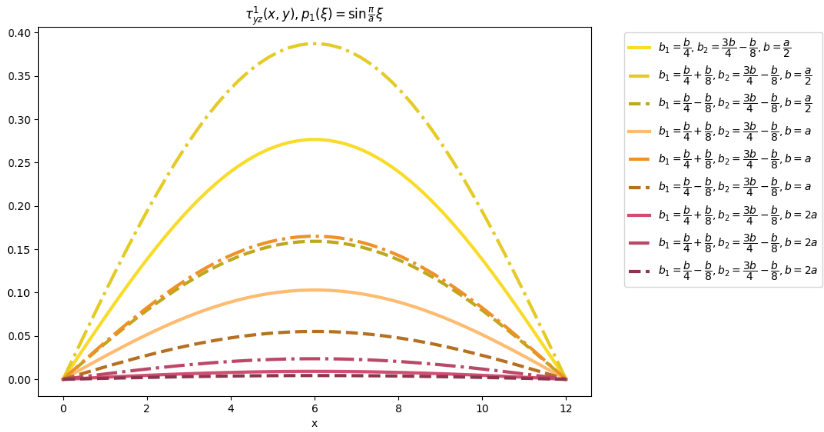


Figure 2.29. Investigation of the geometry of the region G_1

For the second layer:

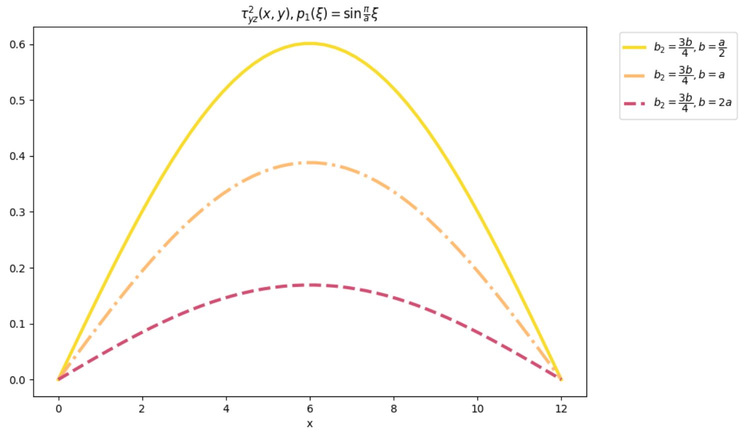


Figure 2.30. Investigation of the geometry of the region G_2

A more advanced dependency:

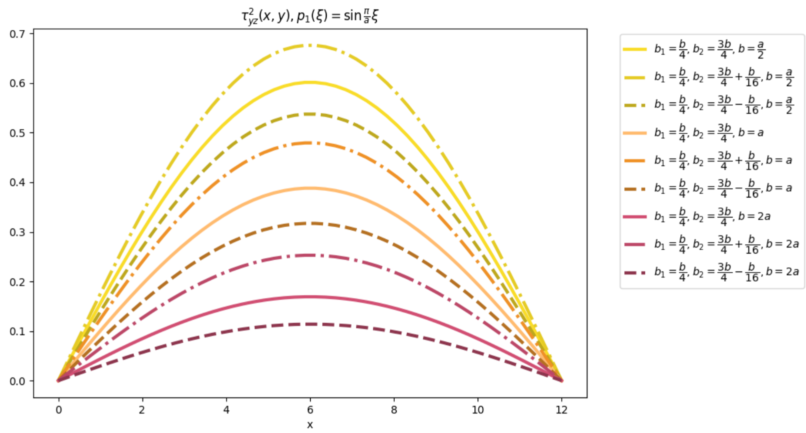


Figure 2.31. Investigation of the geometry of the region G_2

As can be seen from the graphs (Fig.2.30)"— (Fig.2.31) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.
It is also interesting to consider the dependence of stress on layer thicknesses:

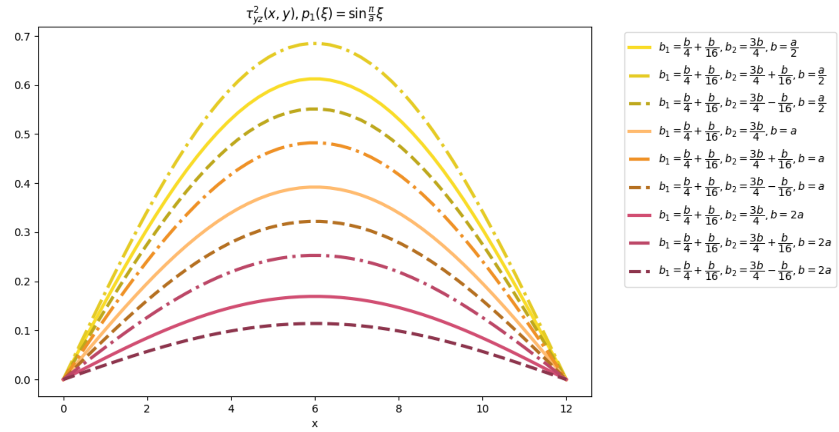


Figure 2.32. Investigation of the geometry of the region G_2

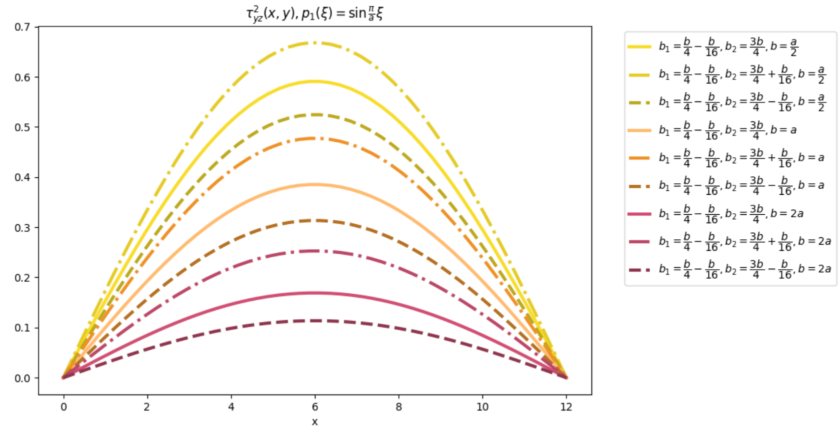


Figure 2.33. Investigation of the geometry of the region G_2

For the third layer:

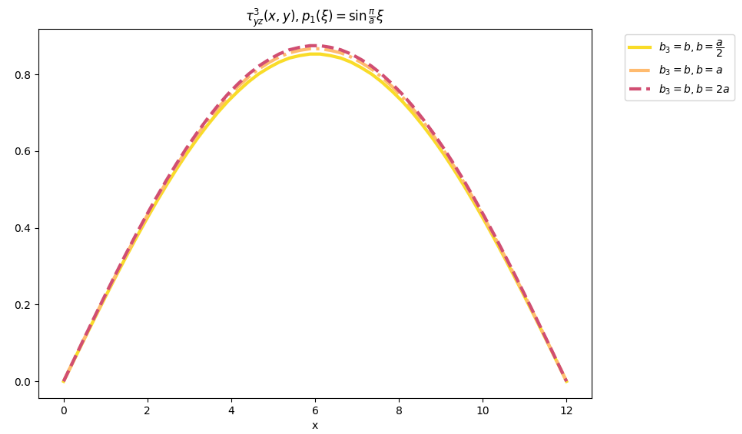


Figure 2.34. Investigation of the geometry of the region G_3

A more advanced dependency:

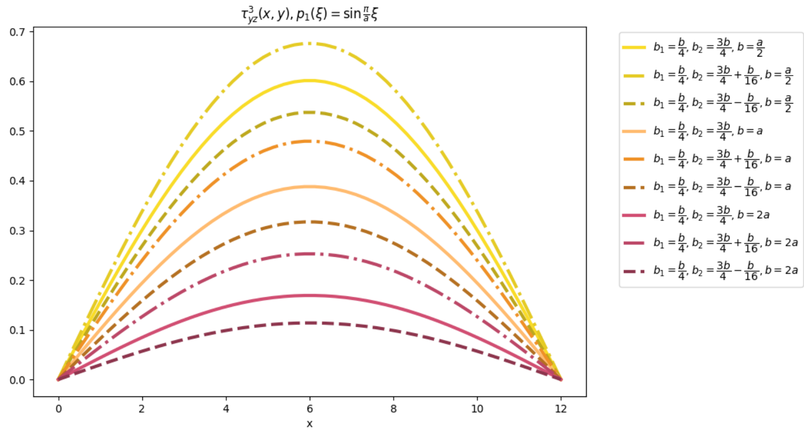


Figure 2.35. Investigation of the geometry of the region G_3

As can be seen from the graphs (Fig.2.34)"— (Fig.2.35) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.
It is also interesting to consider the dependence of stress on layer thicknesses:

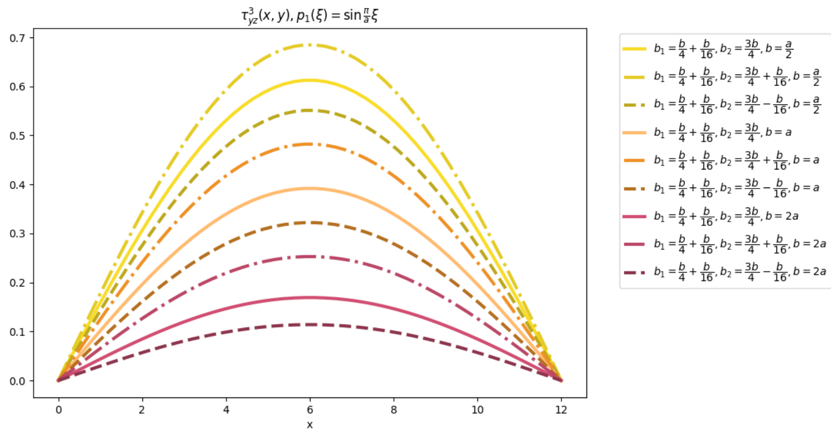


Figure 2.36. Investigation of the geometry of the region G_3

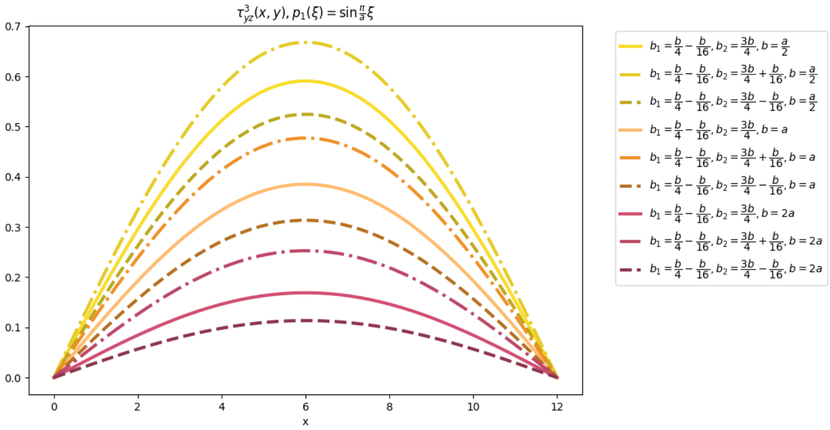


Figure 2.37. Investigation of the geometry of the region G_3

2.5.6. Dependence of the stress value on the arrangement of materials G_{213}

It is also necessary to consider the issue of the dependence of stress values on the material on which a load of intensity $p(x)$ acts Calculations were made with the following parameters:
Area parameters:

- $a = 12$
- $b = 12$
- $b_1 = 3$
- $b_2 = 9$
- $G_1 = 4.0 \cdot 10^{10}$ "— rolled manganese bronze
- $G_2 = 8.0 \cdot 10^{10}$ "— carbon steel
- $G_3 = 2.7 \cdot 10^{10}$ "— duralumin

Load parameters:

- $p_1(\xi) = \sin\frac{\pi}{a}\xi$

Consider the dependence of stress on the geometry of a two-layer rectangular region.
For the first layer:

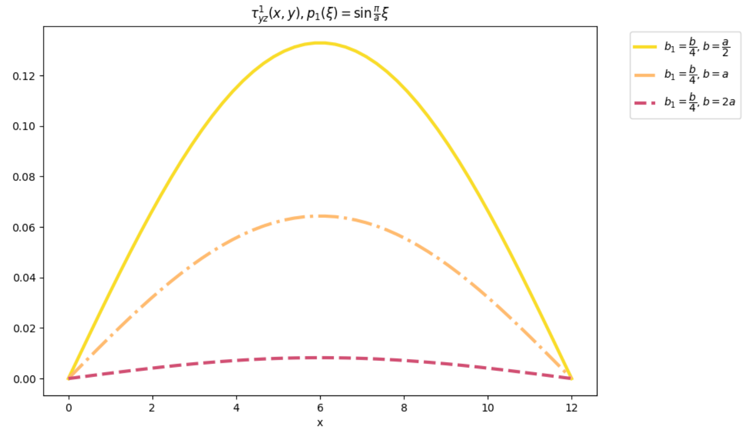


Figure 2.38. Investigation of the geometry of the region G_1

A more advanced dependency:

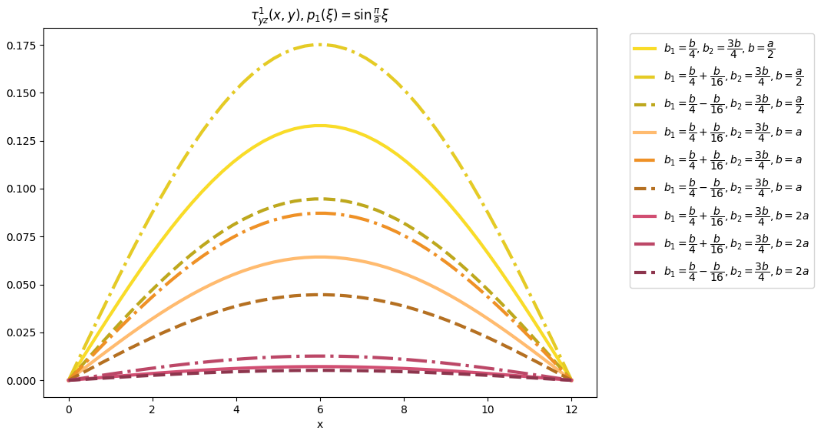


Figure 2.39. Investigation of the geometry of the region G_1

As can be seen from the graphs (Fig.2.38)"— (Fig.2.39) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.
It is also interesting to consider the dependence of stress on layer thicknesses:

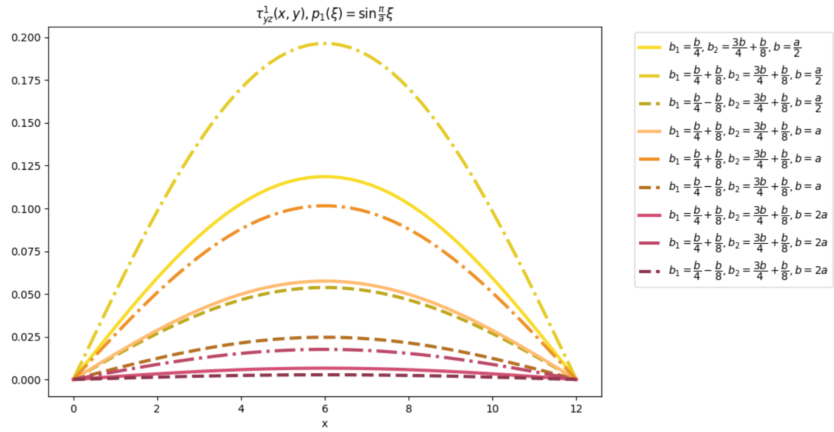


Figure 2.40. Investigation of the geometry of the region G_1

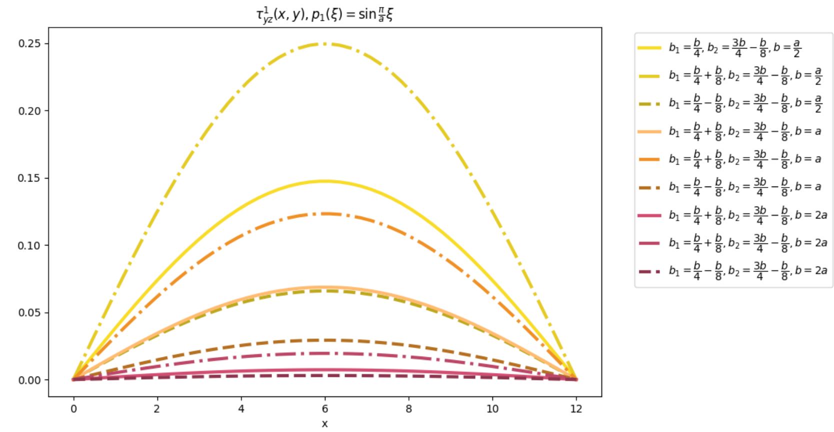


Figure 2.41. Investigation of the geometry of the region G_1

For the second layer:

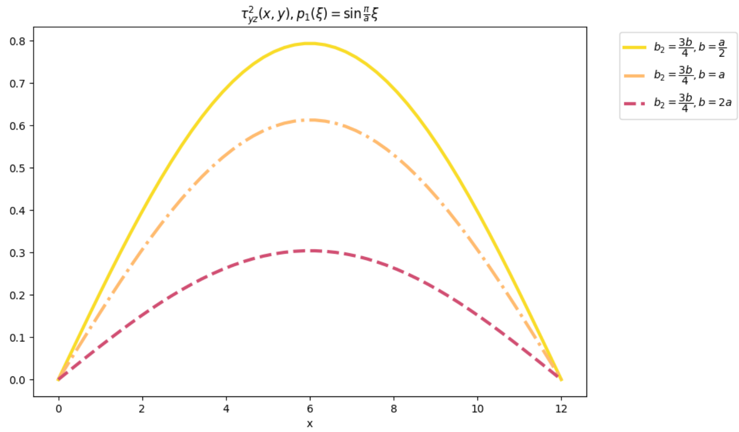


Figure 2.42. Investigation of the geometry of the region G_2

A more advanced dependency:

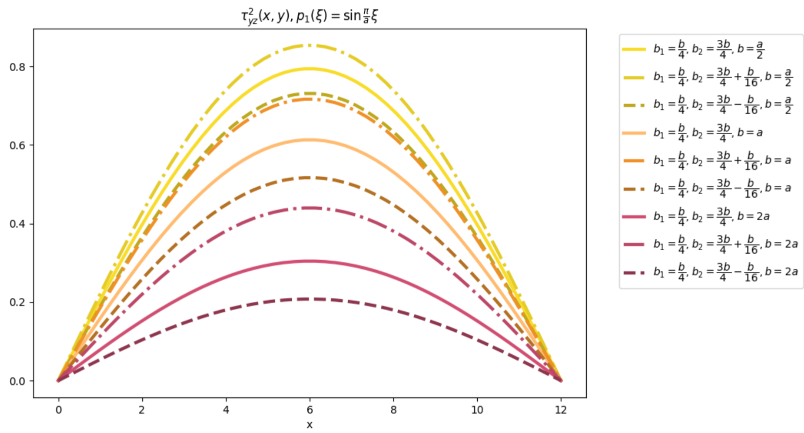


Figure 2.43. Investigation of the geometry of the region G_2

As can be seen from the graphs (Fig.2.42)"— (Fig.2.43) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.
It is also interesting to consider the dependence of stress on layer thicknesses:

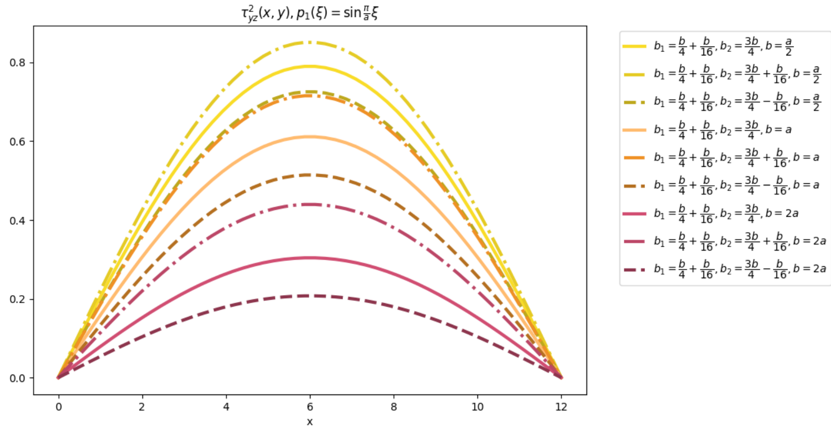


Figure 2.44. Investigation of the geometry of the region G_2

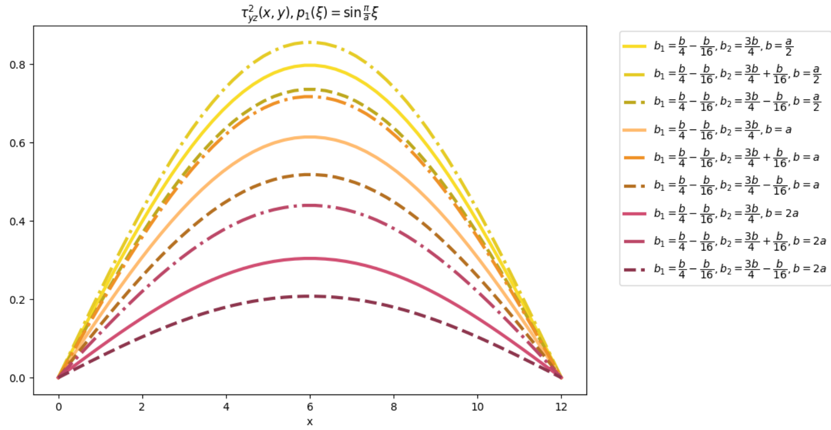


Figure 2.45. Investigation of the geometry of the region G_2

For the third layer:

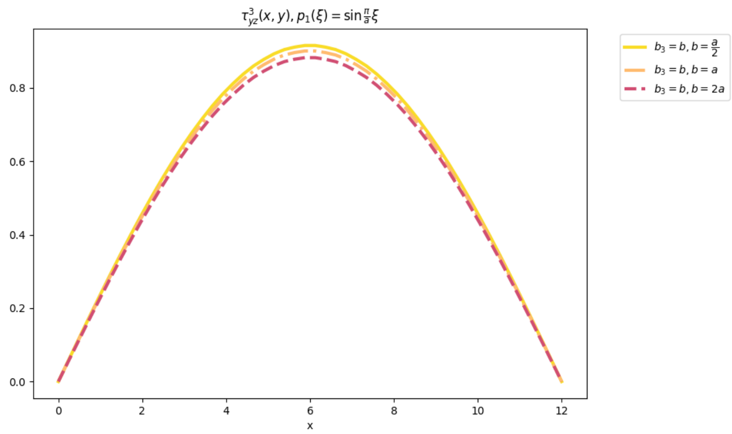


Figure 2.46. Investigation of the geometry of the region G_3

A more advanced dependency:

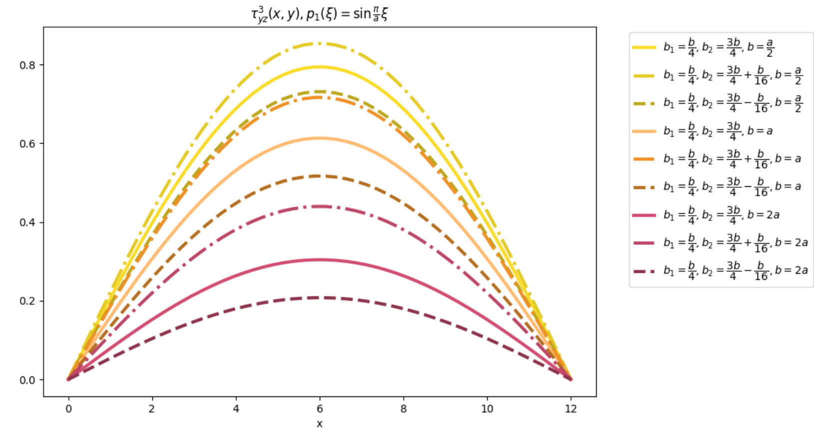


Figure 2.47. Investigation of the geometry of the region G_3

As can be seen from the graphs (Fig.2.46)"— (Fig.2.47) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.
It is also interesting to consider the dependence of stress on layer thicknesses:

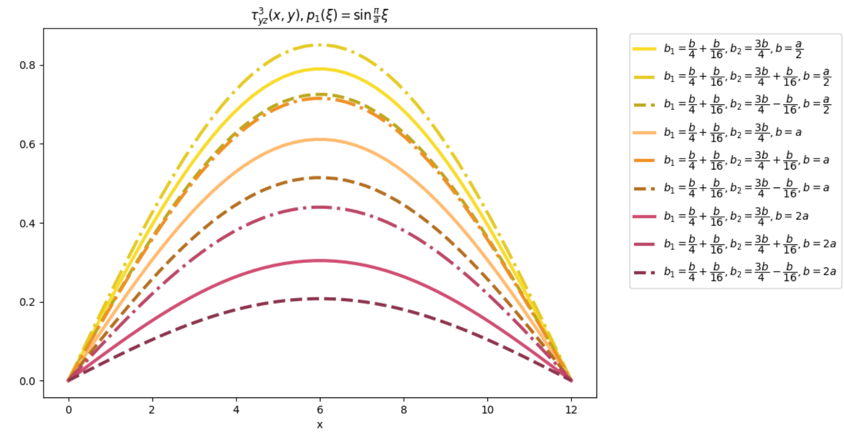


Figure 2.48. Investigation of the geometry of the region G_3

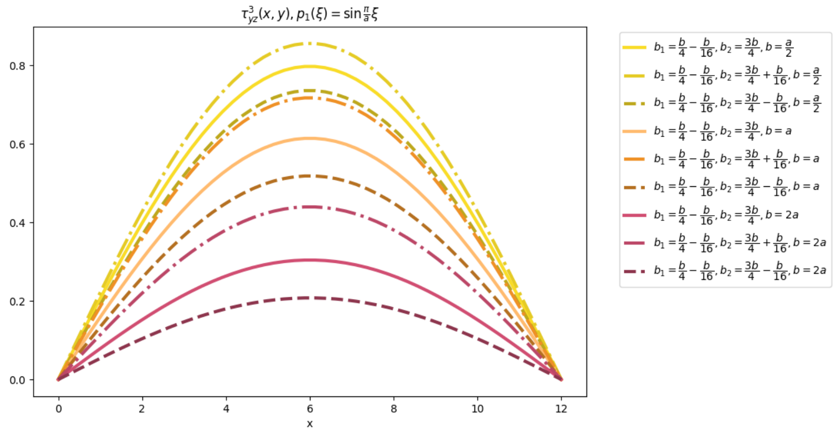


Figure 2.49. Investigation of the geometry of the region G_3

2.5.7. Dependence of the stress value on the arrangement of materials G_{231}

It is also necessary to consider the issue of the dependence of stress values on the material on which a load of intensity $p\left(x\right)$ acts Calculations were made with the following parameters:
Area parameters:

- $a = 12$
- $b = 12$
- $b_1 = 3$
- $b_2 = 9$
- $G_1 = 4.0 \cdot 10^{10}$ "— rolled manganese bronze
- $G_2 = 2.7 \cdot 10^{10}$ "— duralumin
- $G_3 = 8.0 \cdot 10^{10}$ "— carbon steel

Load parameters:

- $p_1\left(\xi\right) = \sin\dfrac{\pi}{a}\xi$

Consider the dependence of stress on the geometry of a two-layer rectangular region.
For the first layer:

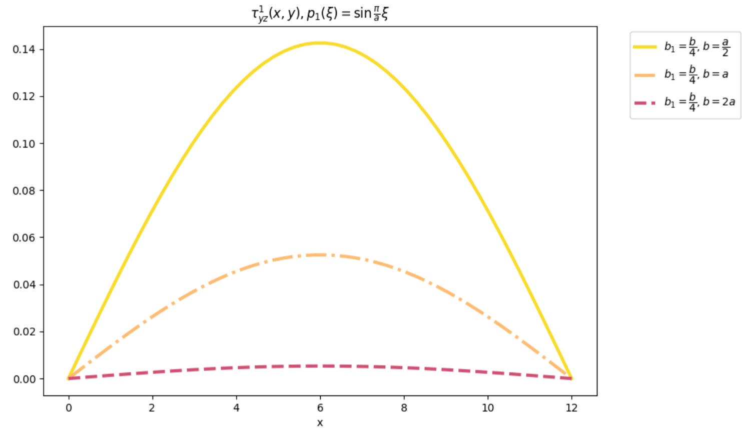


Figure 2.50. Investigation of the geometry of the region G_1

A more advanced dependency:

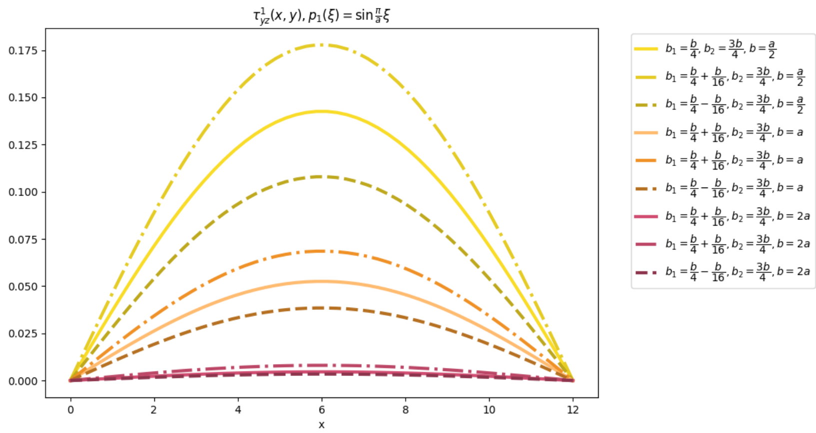


Figure 2.51. Investigation of the geometry of the region G_1

As can be seen from the graphs (Fig.2.50)"— (Fig.2.51) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.
It is also interesting to consider the dependence of stress on layer thicknesses:

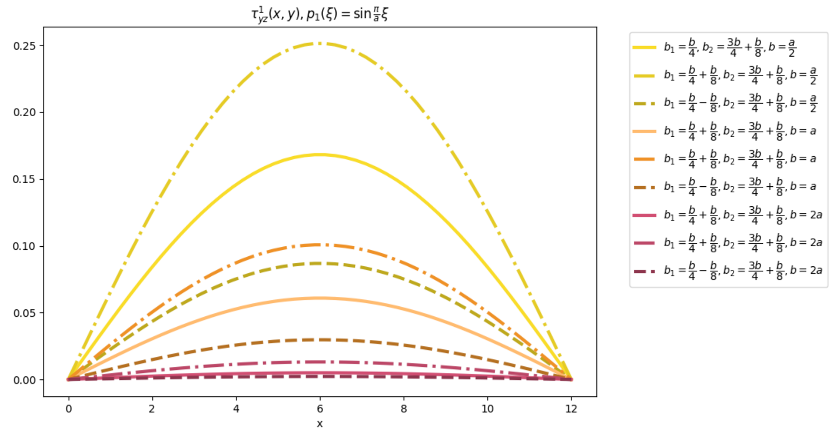


Figure 2.52. Investigation of the geometry of the region G_1

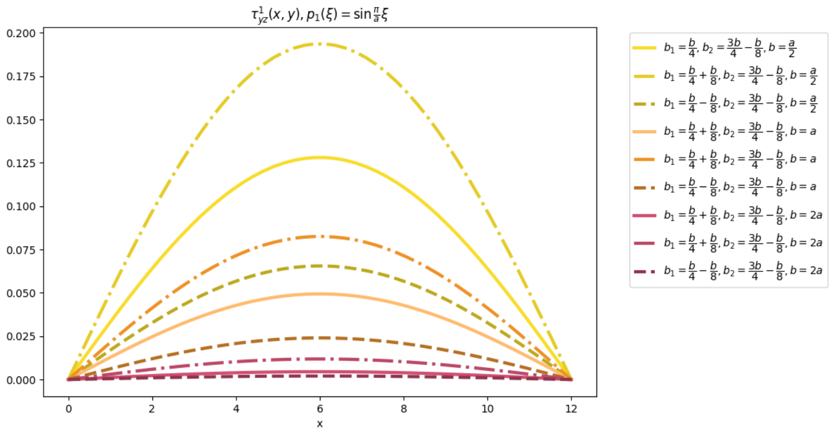


Figure 2.53. Investigation of the geometry of the region G_1

For the second layer:

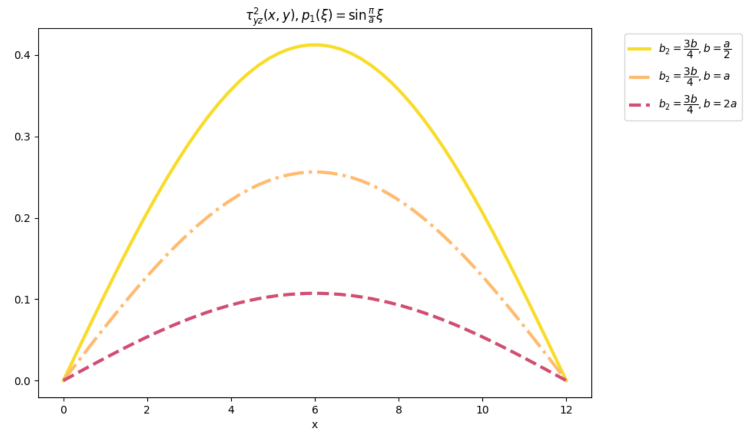


Figure 2.54. Investigation of the geometry of the region G_2

A more advanced dependency:

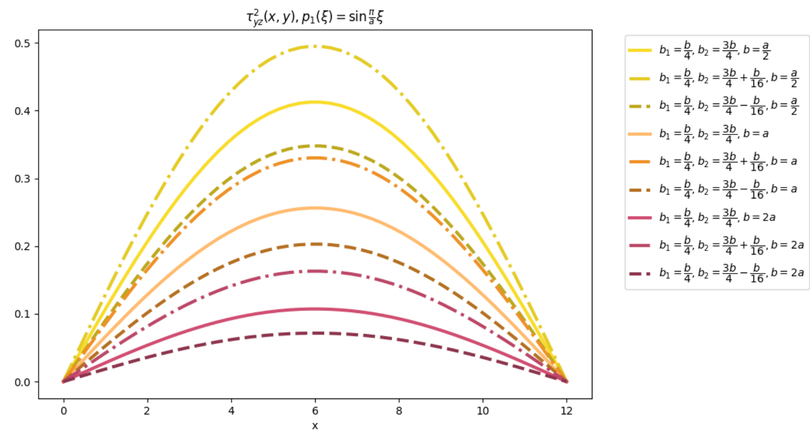


Figure 2.55. Investigation of the geometry of the region G_2

As can be seen from the graphs (Fig.2.54)"— (Fig.2.55) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.
It is also interesting to consider the dependence of stress on layer thicknesses:

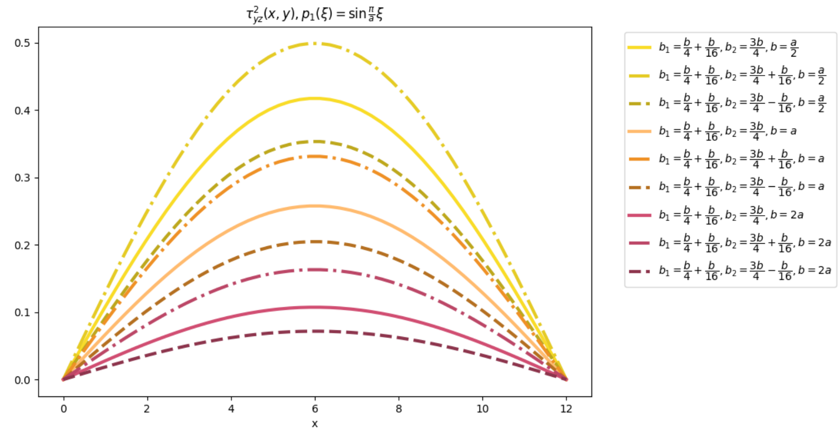


Figure 2.56. Investigation of the geometry of the region G_2

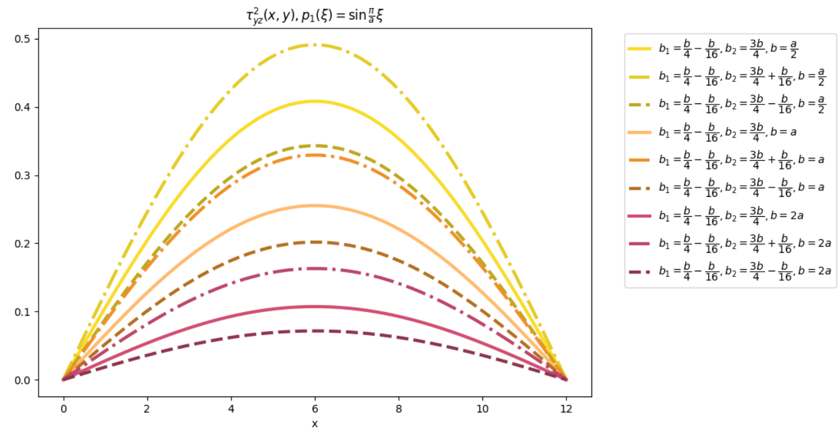


Figure 2.57. Investigation of the geometry of the region G_2

For the third layer:

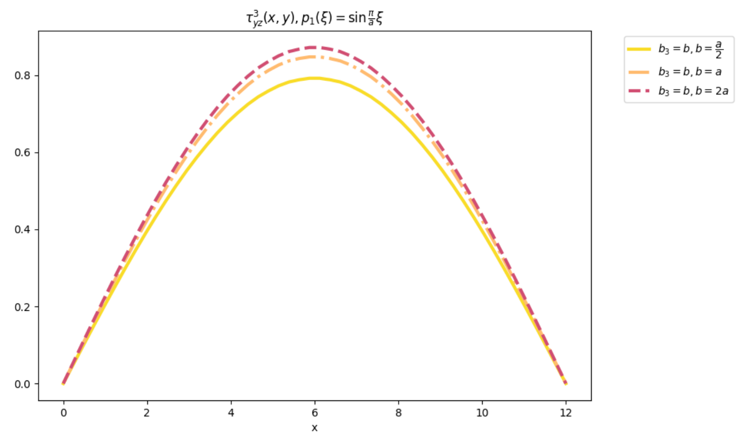


Figure 2.58. Investigation of the geometry of the region G_3

A more advanced dependency:

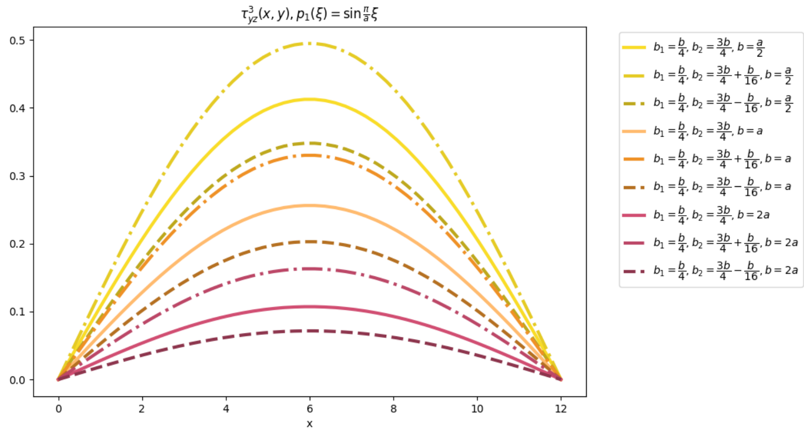


Figure 2.59. Investigation of the geometry of the region G_3

As can be seen from the graphs (Fig.2.58)"— (Fig.2.59) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.

It is also interesting to consider the dependence of stress on layer thicknesses:

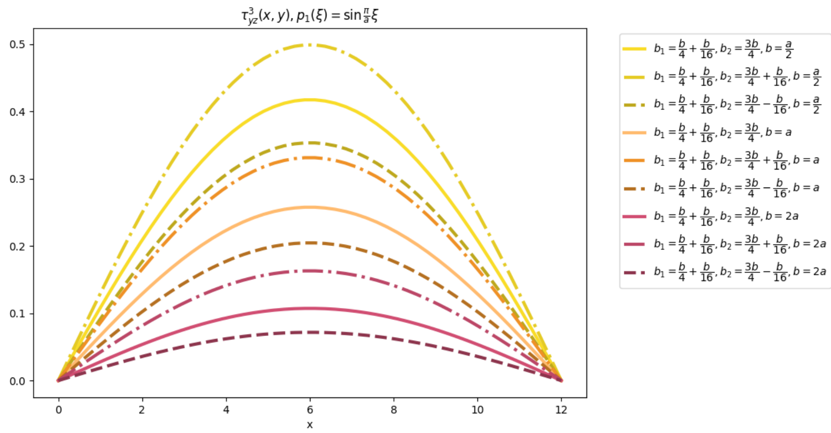


Figure 2.60. Investigation of the geometry of the region G_3

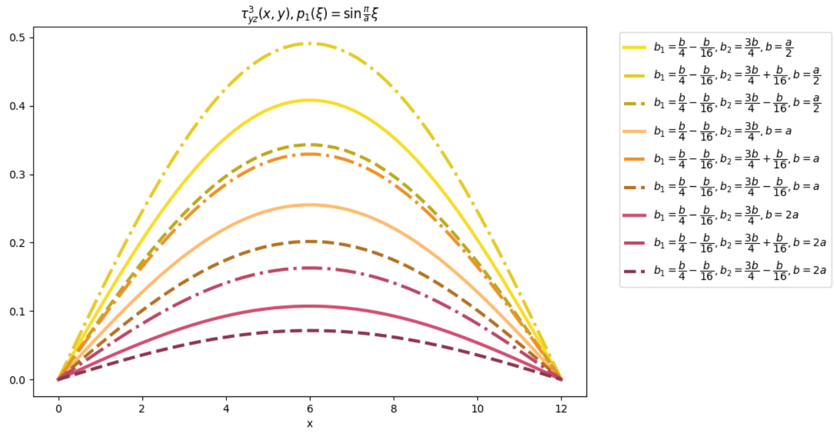


Figure 2.61. Investigation of the geometry of the region G_3

2.5.8. Dependence of the stress value on the arrangement of materials G_{321}

It is also necessary to consider the issue of the dependence of stress values on the material on which a load of intensity $p(x)$ acts Calculations were made with the following parameters:

Area parameters:

- $a = 12$
- $b = 12$
- $b_1 = 3$
- $b_2 = 9$
- $G_1 = 4.0 \cdot 10^{10}$ "— rolled manganese bronze
- $G_2 = 2.7 \cdot 10^{10}$ "— duralumin
- $G_3 = 8.0 \cdot 10^{10}$ "— carbon steel

Load parameters:

- $p_1(\xi) = \sin\frac{\pi}{a}\xi$

Consider the dependence of stress on the geometry of a two-layer rectangular region.

For the first layer:

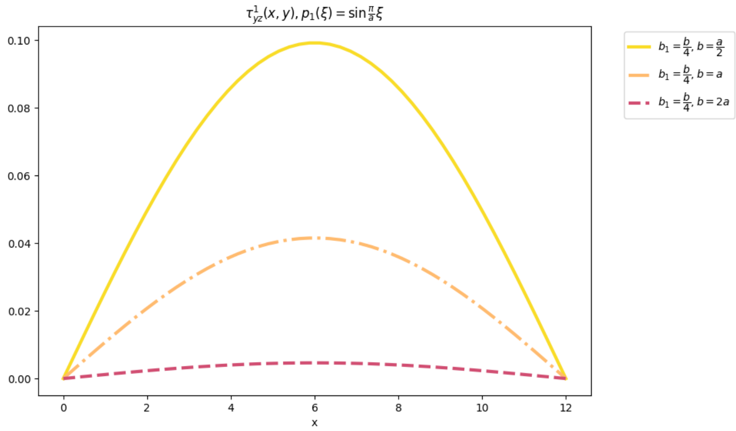


Figure 2.62. Investigation of the geometry of the region G_1

A more advanced dependency:

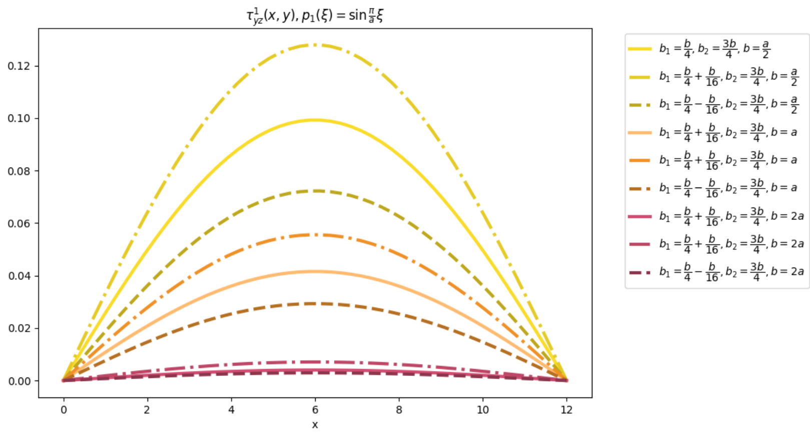


Figure 2.63. Investigation of the geometry of the region G_1

As can be seen from the graphs (Fig.2.62)"— (Fig.2.63) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.

It is also interesting to consider the dependence of stress on layer thicknesses:

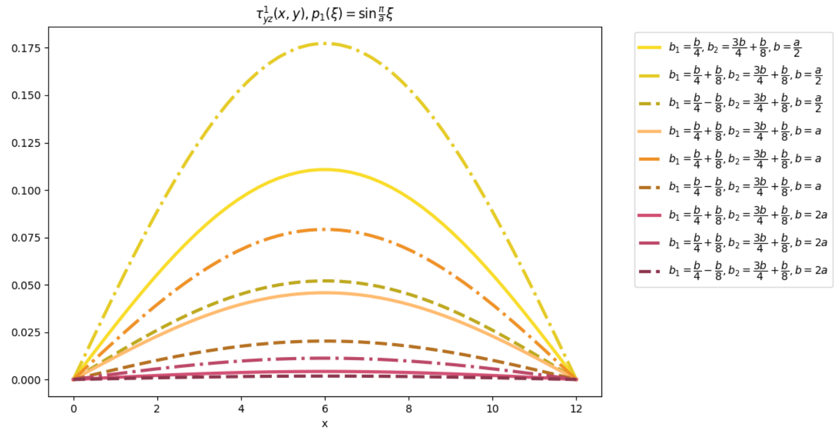


Figure 2.64. Investigation of the geometry of the region G_1

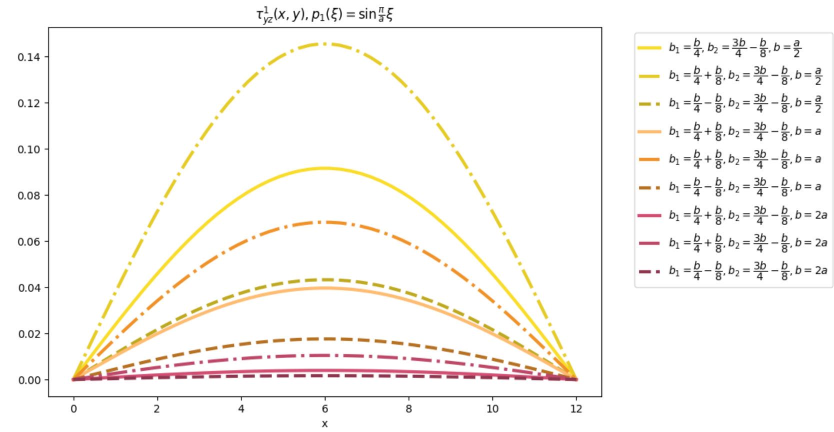


Figure 2.65. Investigation of the geometry of the region G_1

For the second layer:

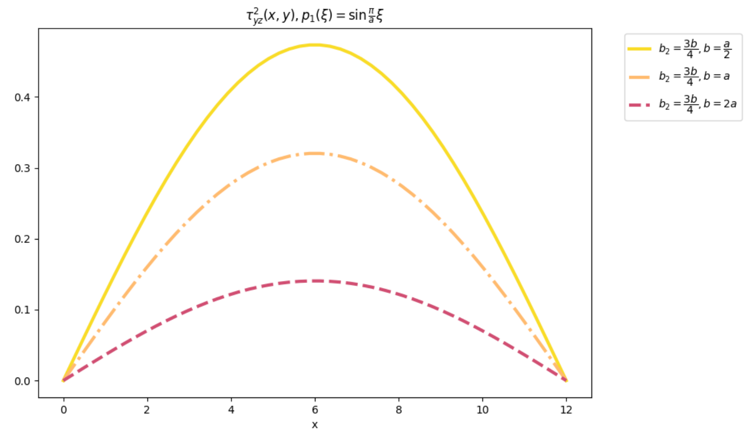


Figure 2.66. Investigation of the geometry of the region G_2

A more advanced dependency:

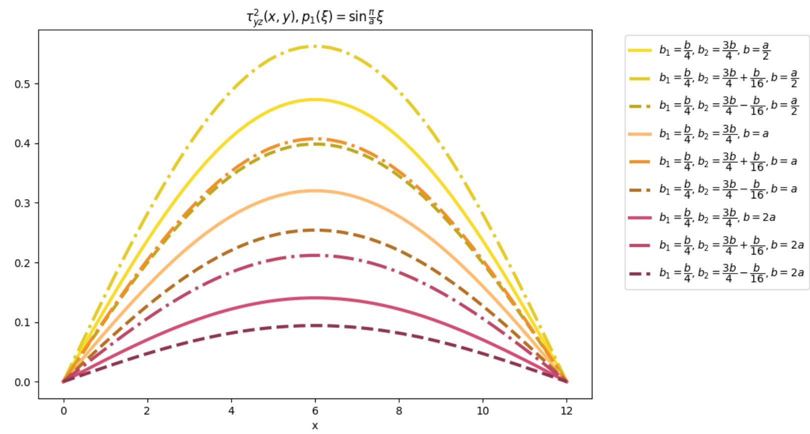


Figure 2.67. Investigation of the geometry of the region G_2

As can be seen from the graphs (Fig.2.66)"— (Fig.2.67) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.

It is also interesting to consider the dependence of stress on layer thicknesses:

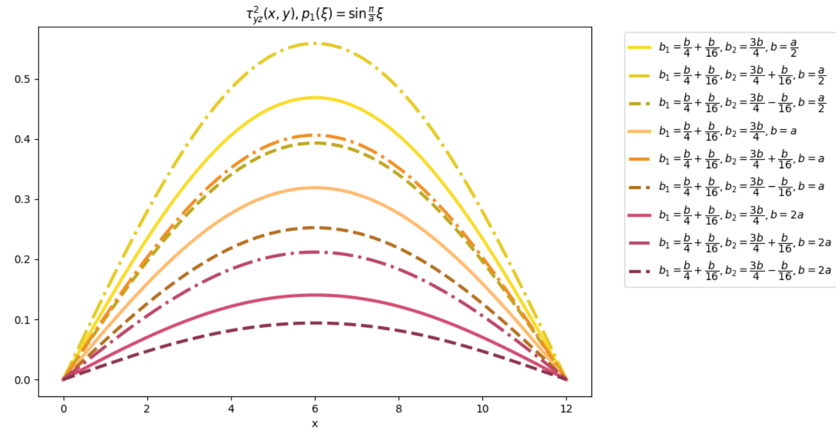


Figure 2.68. Investigation of the geometry of the region G_2

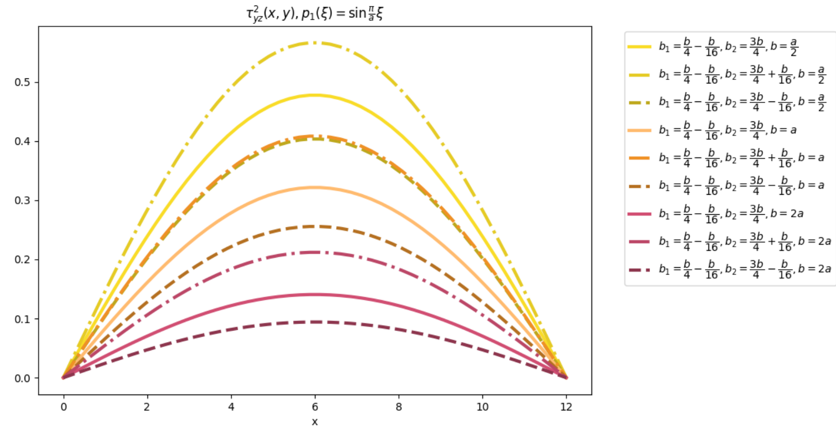


Figure 2.69. Investigation of the geometry of the region G_2

For the third layer:

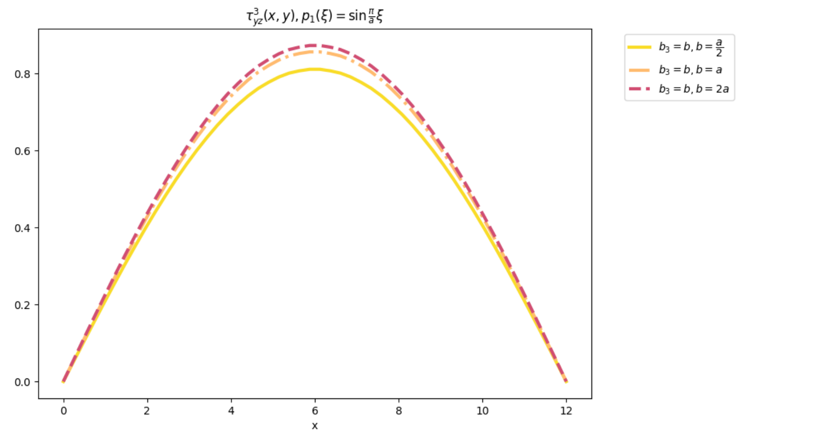


Figure 2.70. Investigation of the geometry of the region G_3

A more advanced dependency:

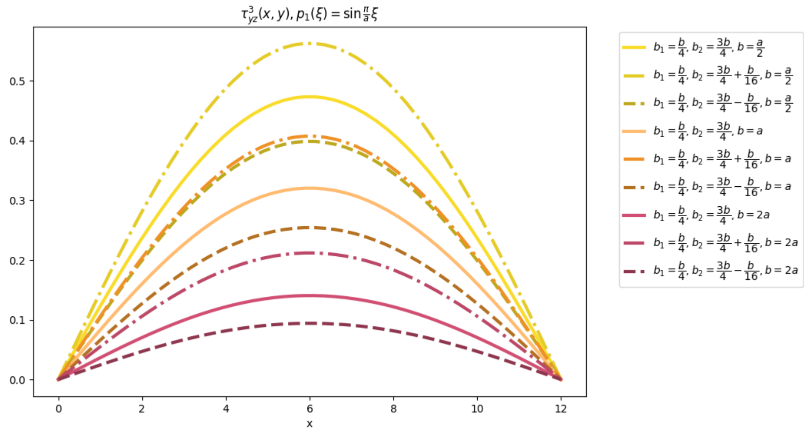


Figure 2.71. Investigation of the geometry of the region G_3

As can be seen from the graphs (Fig.2.70)"— (Fig.2.71) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.
It is also interesting to consider the dependence of stress on layer thicknesses:

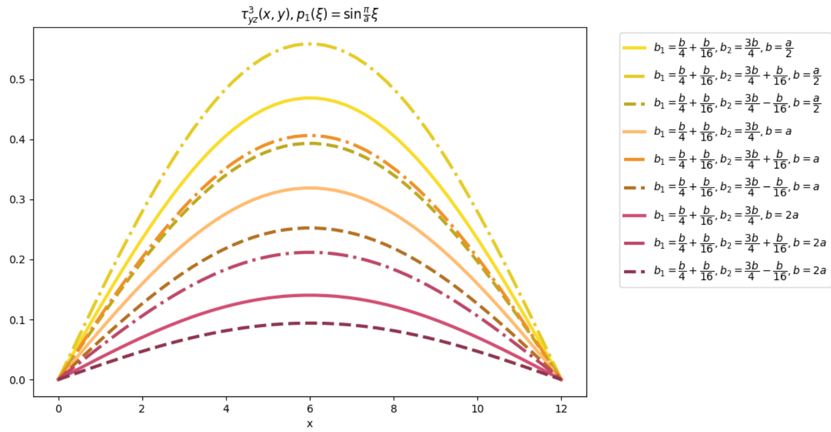


Figure 2.72. Investigation of the geometry of the region G_3

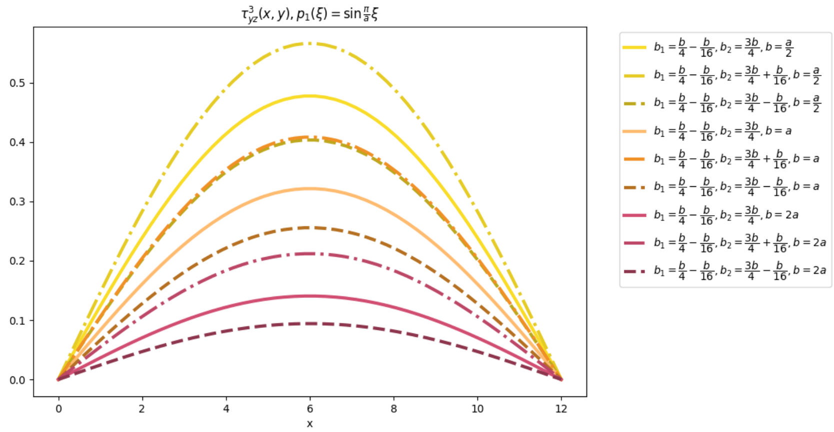


Figure 2.73. Investigation of the geometry of the region G_3

2.5.9. Dependence of the stress value on the arrangement of materials G_{312}

It is also necessary to consider the issue of the dependence of stress values on the material on which a load of intensity $p(x)$ acts Calculations were made with the following parameters:
Area parameters:

- $a = 12$
- $b = 12$
- $b_1 = 3$
- $b_2 = 9$
- $G_1 = 4.0 \cdot 10^{10}$ "— rolled manganese bronze
- $G_2 = 2.7 \cdot 10^{10}$ "— duralumin
- $G_3 = 8.0 \cdot 10^{10}$ "— carbon steel

Load parameters:

- $p_1(\xi) = \sin\frac{\pi}{a}\xi$

Consider the dependence of stress on the geometry of a two-layer rectangular region.
For the first layer:

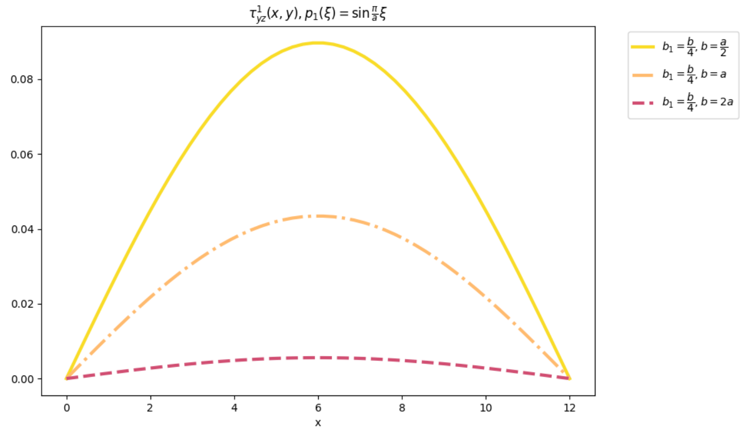


Figure 2.74. Investigation of the geometry of the region G_1

A more advanced dependency:

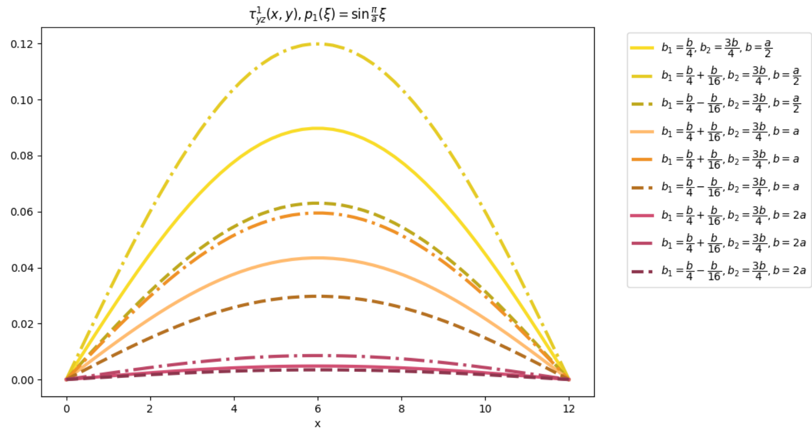


Figure 2.75. Investigation of the geometry of the region G_1

As can be seen from the graphs (Fig.2.74)"— (Fig.**??**) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.
It is also interesting to consider the dependence of stress on layer thicknesses:

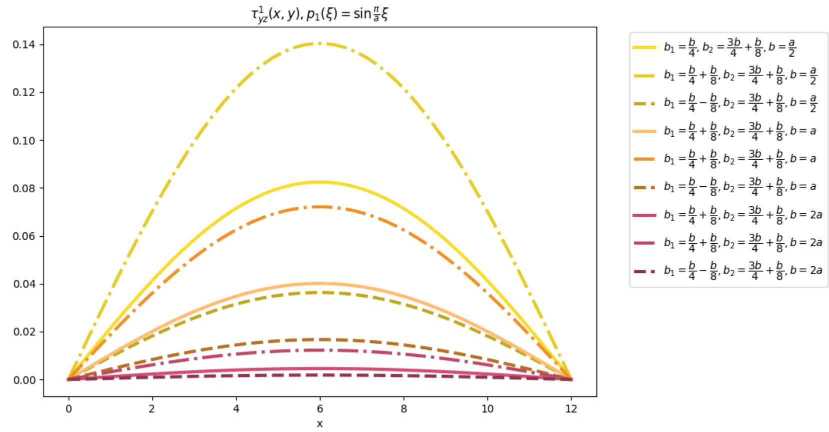


Figure 2.76. Investigation of the geometry of the region G_1

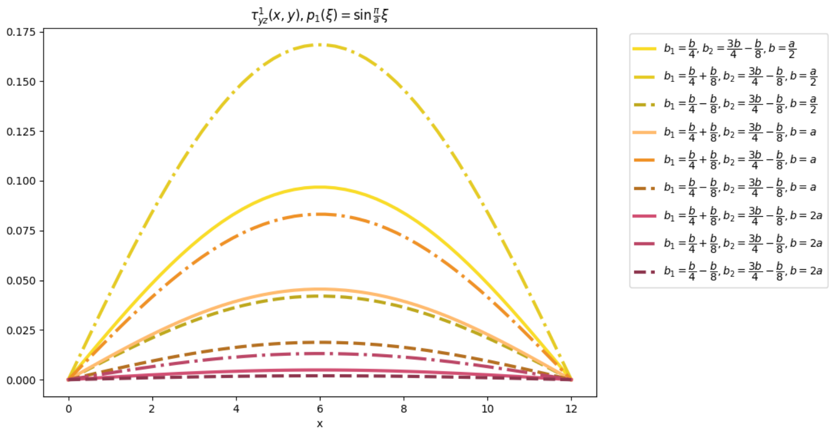


Figure 2.77. Investigation of the geometry of the region G_1

For the second layer:

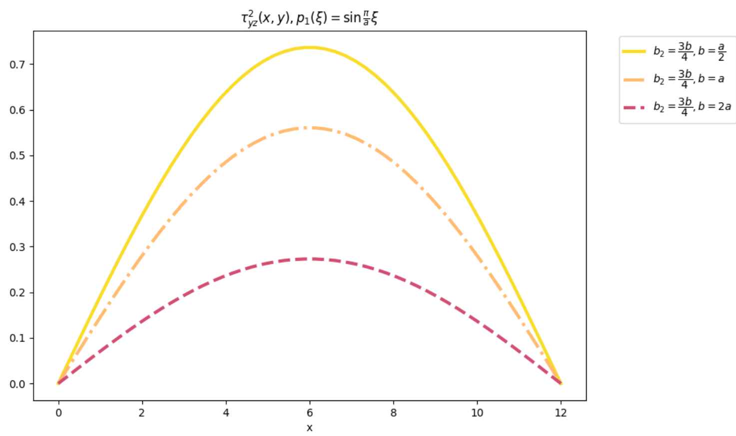


Figure 2.78. Investigation of the geometry of the region G_2

A more advanced dependency:

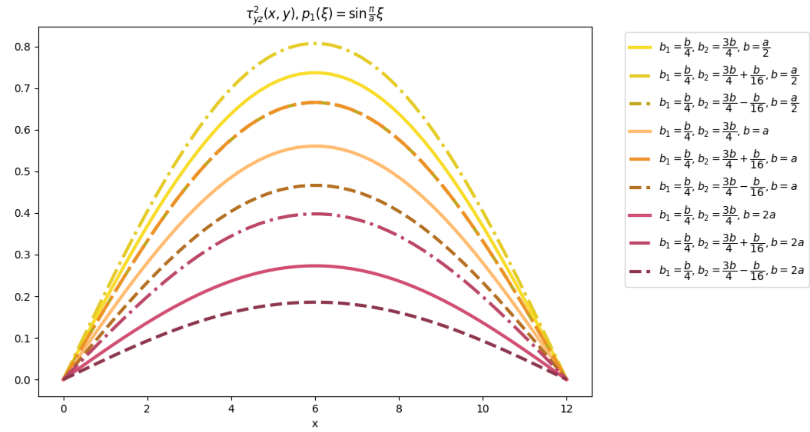


Figure 2.79. Investigation of the geometry of the region G_2

As can be seen from the graphs (Fig.2.78)"— (Fig.2.79) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.

It is also interesting to consider the dependence of stress on layer thicknesses:

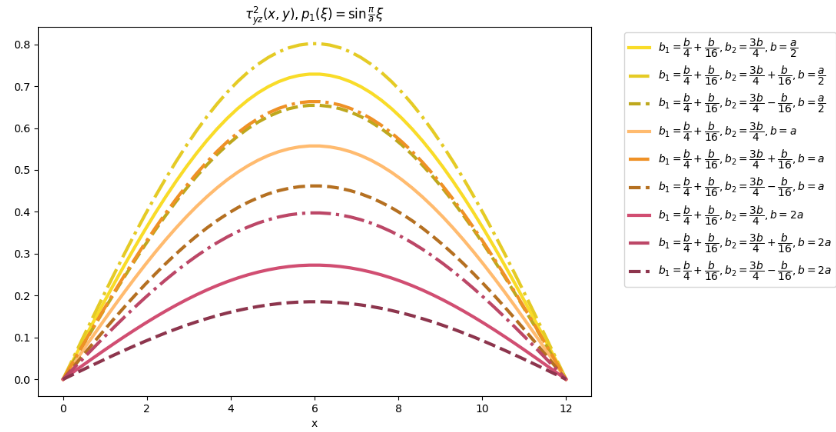


Figure 2.80. Investigation of the geometry of the region G_2

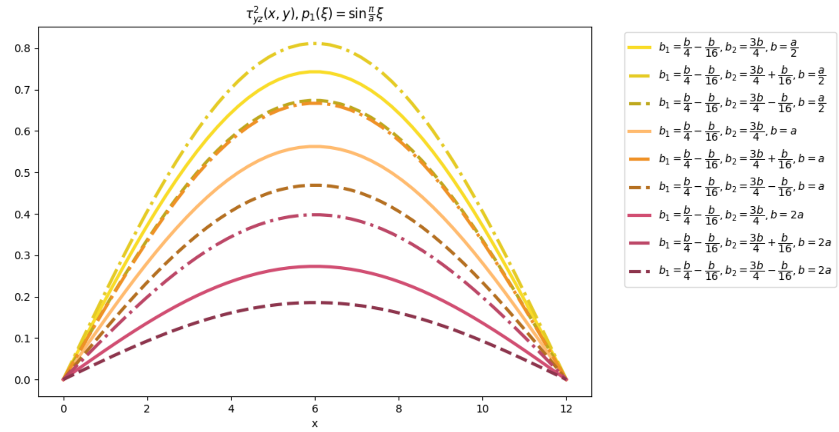


Figure 2.81. Investigation of the geometry of the region G_2

For the third layer:

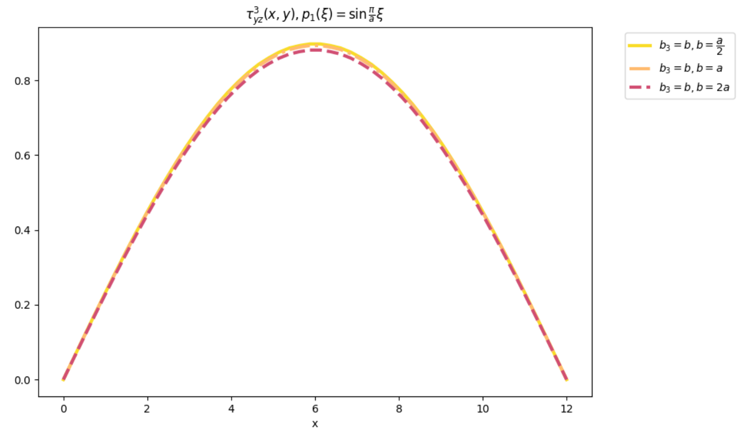


Figure 2.82. Investigation of the geometry of the region G_3

A more advanced dependency:

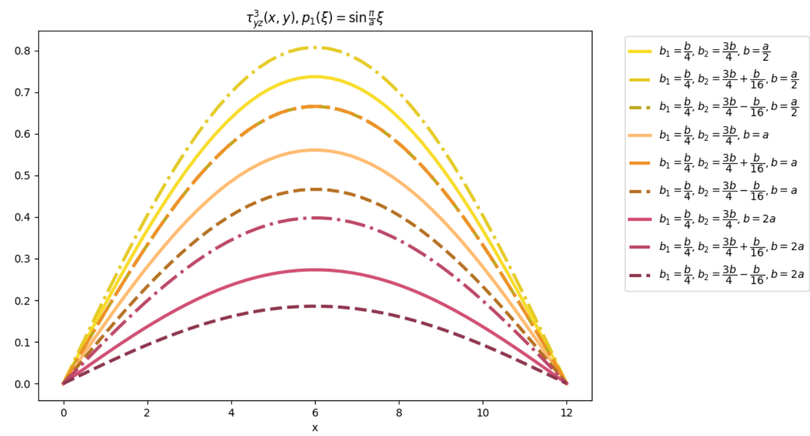


Figure 2.83. Investigation of the geometry of the region G_3

As can be seen from the graphs (Fig.2.82)"— (Fig.2.83) that the stress increases when b_1 is less than b_2,b_2 is less than b and b is less than a.

It is also interesting to consider the dependence of stress on layer thicknesses:

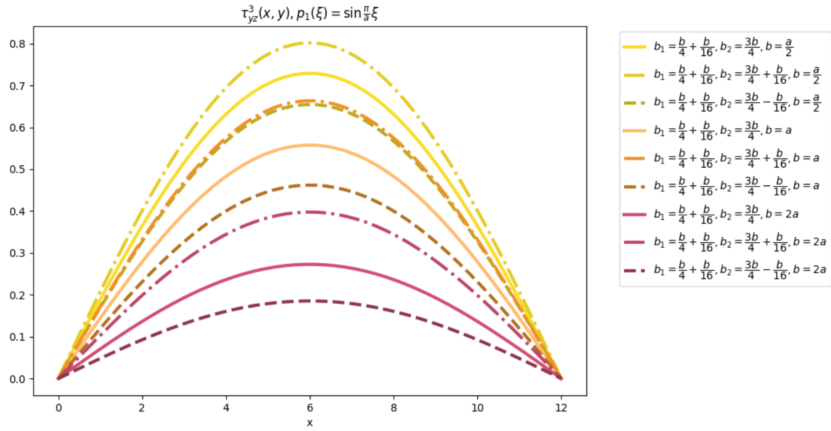


Figure 2.84. Investigation of the geometry of the region G_3

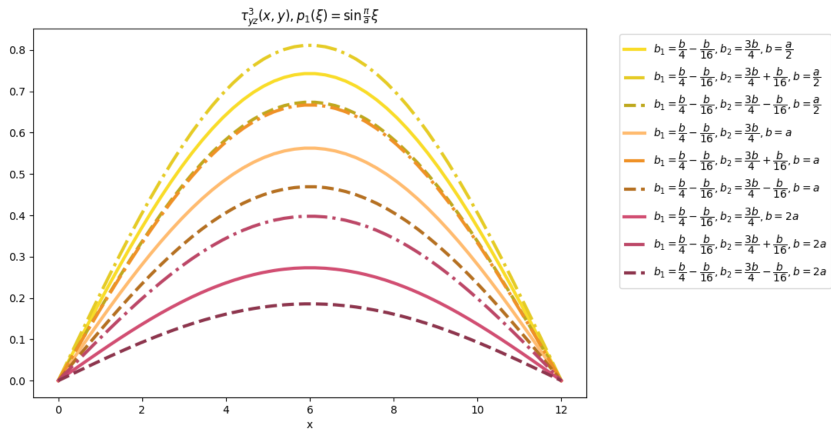


Figure 2.85. Investigation of the geometry of the region G_3

2.6. Conclusions to the second section

The investigated antiplane problem for An N-layered rectangular area under the influence of various types of loads given along the y axis and a particular case when the area consists of three layers

1) The solution of the anti-plane problem of the theory of elasticity for the N-layered rectangular region is constructed using the apparatus of integral transformations.
2) The behaviour of stresses inside a rectangular zone for different aspect ratios has been studied. It is established that the greatest stresses are achieved when the length of the rectangle is greater than its width.
3) The change of stresses when changing materials and load was studied.
4) This approach can be used to construct a solution with interfacial cracks.

CHAPTER 3

THE ANTI-PLANE PROBLEM OF THE THEORY OF ELASTICITY FOR THE MULTILAYERED RECTANGULAR REGION WITH INTERFACIAL CRACKS

3.1. Statement of the problem

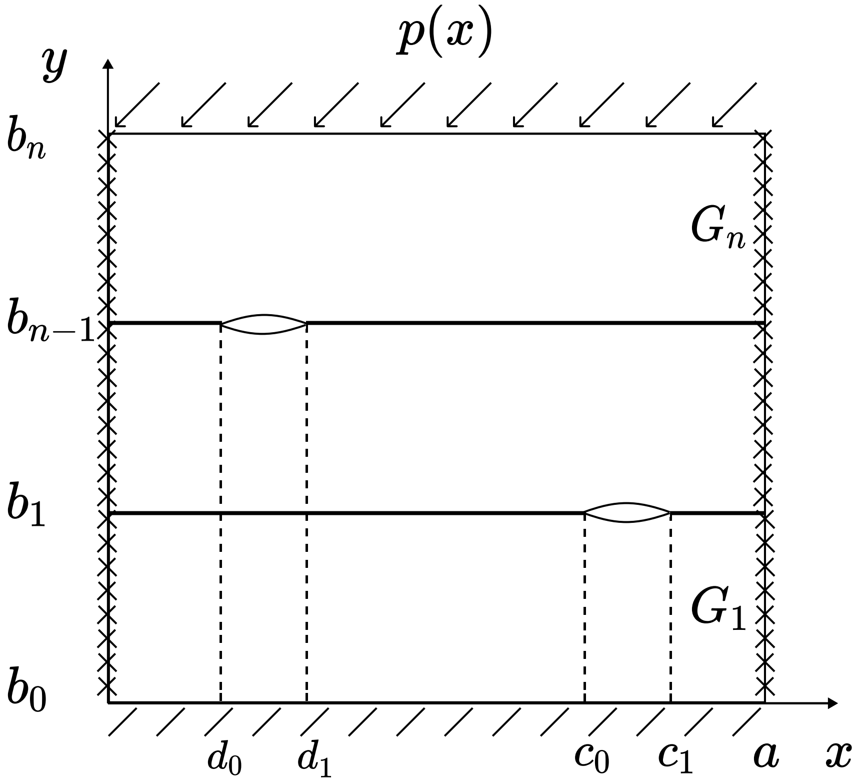


Figure 3.1. Geometry and coordinate system of a rectangular region

The area under consideration (Fig.3.1) (G_N — shear modulus of the k-th layer) occupies the area described in the Cartesian coordinate system by the relations $0 < x < a, b_{k-1} < y < b_k$, which is in a state of anti-plane deformation. This region is divided into N heterogeneous layers along the y axis. Let the edges $x = 0$,$x = a$ be immovably fixed.

$$W_k\Big|_{x=0} = 0,\ \ W_k\Big|_{x=a} = 0,\ \ b_{k-1} < y < b_k,\ \ k = \overline{1,N} \tag{3.1}$$

where $W_k\,(x,y)$— movement relative to the z axis in the k-th layer, $b_0 = 0$,$b_N = b$. The face $y = 0$ is in smooth contact conditions, the face $y = b$ is subjected to a load of intensity $p\,(x)$

$$\tau_{yz}^{1}\Big|_{y=0} = 0,\ \ \tau_{yz}^{N}\Big|_{y=b_N} = p\,(x)\,, 0 < x < a \tag{3.2}$$

where $\tau_{yz}^{1}\,(x,y)\,$,$\tau_{yz}^{N}\,(x,y)$— tangential stresses of the first and Nth layers, respectively .

Conjugation conditions and crack conditions are fulfilled between the layers:

$$\begin{aligned} W_k\Big|_{y\,=\,b_k\,-\,0} &= W_{k+1}\Big|_{y\,=\,b_k\,+\,0} + \chi_k(x), \\ & \qquad\qquad 0 < x < a \\ \tau_{yz}^{k}\Big|_{y\,=\,b_k\,-\,0} &= \tau_{yz}^{k+1}\Big|_{y\,=\,b_k\,+\,0} = \zeta_k(x) \end{aligned} \tag{3.3}$$

where:

$$\chi_k(x) = \begin{cases} \neq 0, & x \in [c_k - 1; c_k] \\ 0, & x \in [0; c_{k-1}) \cup (c_1; a] \end{cases} ,k = \overline{1,N}$$

It is necessary to find the displacement and stress of each of the layers satisfying the conditions (3.1)— (3.3) and the equilibrium equation:

$$\frac{\partial^2 W_k}{\partial x^2} + \frac{\partial^2 W_k}{\partial y^2} = 0,\ \ 0 < x < a,\ \ b_{k-1} < y < b_k \tag{3.4}$$

Write down the boundary value problem

$$\begin{cases} \dfrac{\partial^2 W_k}{\partial x^2} + \dfrac{\partial^2 W_k}{\partial y^2} = 0, \qquad 0 < x < a, \qquad b_{k-1} < y < b_k \\ W_k\Big|_{x=0} = 0, \qquad W_k\Big|_{x=a} = 0 \qquad b_{k-1} < y < b_k \\ \tau_{yz}^1\Big|_{y=0} = 0, \qquad \tau_{yz}^N\Big|_{y=b_N} = p\left(x\right), \quad 0 < x < a, \quad b_{k-1} < y < b_k \end{cases} \tag{3.5}$$

where:

$$\tau_{yz}^1\Big|_{y=0} = 0 \Leftrightarrow \frac{\partial W_1}{\partial y}\Big|_{y=0} = 0, \ \ \tau_{yz}^N\Big|_{y=b_N} = p(x) \Leftrightarrow \frac{\partial W_N}{\partial y}\Big|_{y=b_N} = \frac{p(x)}{G_N} \tag{3.6}$$

Conjugation conditions:

$$\begin{aligned} W_k\Big|_{y\,=\,b_k-0} &= W_k\Big|_{y\,=\,b_k+0} + \chi_k(x), \\ & \qquad\qquad 0 < x < a \\ \tau_{yz}^k\Big|_{y\,=\,b_k-0} &= \tau_{yz}^{k+1}\Big|_{y\,=\,b_k+0} = \zeta_k(x) \end{aligned} \tag{3.7}$$

where:

$$\tau_{yz}^k\Big|_{y=b_k-0} = \tau_{yz}^{k+1}\Big|_{y=b_k+0} \Leftrightarrow G_k \frac{\partial W_k}{\partial y}\Big|_{y=b_k-0} = G_{k+1}\frac{\partial W_{k+1}}{\partial y}\Big|_{y=b_k+0}$$

G_k — shear modulus of the k-th layer, $k = \overline{1,N}$

3.2. Reducing the initial problem to the one-dimensional problem

The input problem was reduced to a one-dimensional one using the finite integral sin Fourier transform of the variable x:

$$W_{\alpha_n,k}(y) = \int_0^a W_k(x,y) \sin \alpha_n x \ \ \mathrm{d}x \tag{3.8}$$

With the inversion formula:

$$W_k(x,y) = \frac{2}{a} \sum_{n=0}^{\infty} W_{\alpha_n,k}(y) \sin \alpha_n x \tag{3.9}$$

Write down the boundary conditions in the transform domain:

$$W^{'}_{\alpha_n,1}\bigg|_{y=0} = \int_0^a \frac{\partial W_1}{\partial y}\bigg|_{y=0} \sin \alpha_n x \ \ \mathrm{d}x \tag{3.10}$$

$$W^{'}_{\alpha_n,N}\bigg|_{y=b_N} = \int_0^a \frac{\partial W_N}{\partial y}\bigg|_{y=b_N} \sin \alpha_n x \ \ \mathrm{d}x \tag{3.11}$$

Write the load in the transform domain:

$$\frac{p_{\alpha_n}}{G_N} = \int_0^a \frac{p(x)}{G_N} \sin \alpha_n x \ \ \mathrm{d}x \tag{3.12}$$

Write down the conjugation conditions in the transform domain:

$$W_{\alpha_n,k}\bigg|_{y=b_k-0} = \int_0^a W_k\bigg|_{y=b_k-0} \sin \alpha_n x \ ; \ \mathrm{d}x \tag{3.13}$$

$$W_{\alpha_n,k+1}\bigg|_{y=b_k+0} = \int_0^a W_{k+1}\bigg|_{y=b_k+0} \sin \alpha_n x \ \ \mathrm{d}x \tag{3.14}$$

$$G_k W^{'}_{\alpha_n,k}\bigg|_{y=b_k-0} = G_k \int_0^a \frac{\partial W_k}{\partial y}\bigg|_{y=b_k-0} \sin \alpha_n x \ \ \mathrm{d}x \tag{3.15}$$

$$G_{k+1} W^{'}_{\alpha_n,k+1}\bigg|_{y=b_k+0} = G_{k+1} \int_0^a \frac{\partial W_{k+1}}{\partial y}\bigg|_{y=b_k+0} \sin \alpha_n x \ \ \mathrm{d}x \tag{3.16}$$

$$\chi_{\alpha_n,k} = \int_0^a \chi_k(x) \sin \alpha_n x \, dx \tag{3.17}$$

$$\zeta_{\alpha_n,k} = \int_0^a \zeta_k(x) \sin \alpha_n x \, dx \tag{3.18}$$

Write down the boundary value problem:

$$\begin{cases} W''_{\alpha_n,k}(y) - \alpha_n^2 W_{\alpha_n,k}(y) = 0 \\ W'_{\alpha_n,1}\Big|_{y=0} = 0, \quad W'_{\alpha_n,N}\Big|_{y=b} = \dfrac{p_{\alpha_n}}{G_N} \\ W_{\alpha_n,k}\Big|_{y=b_k-0} = W_{\alpha_n,k+1}\Big|_{y=b_k+0} + \chi_{\alpha_n,k} \\ G_k W'_{\alpha_n,k}\Big|_{y=b_k-0} = G_{k+1} W'_{\alpha_n,k+1}\Big|_{y=b_k+0} = \zeta_{\alpha_n,k} \end{cases} \tag{3.19}$$

where α_n — integral transform parameter,, G_N — shear modulus of the N-th layer

The general solutions (3.19) will then be found in the form:

$$\begin{aligned} W_{\alpha_n,k}(y) &= A_k e^{\alpha_n y} + B_k e^{-\alpha_n y} + \chi_{\alpha_n,k} \\ W'_{\alpha_n,k}(y) &= \alpha_n \left(A_k e^{\alpha_n y} - B_k e^{-\alpha_n y} \right) \end{aligned} \tag{3.20}$$

where A_k,B_k — unknown constants.

Using conjugation conditions:

$$\begin{cases} A_k e^{\alpha_n b_k} + B_k e^{-\alpha_n b_k} = A_{k+1} e^{\alpha_n b_k} + B_{k+1} e^{-\alpha_n b_k} + \chi_{\alpha_n,k} \\ G_k(A_k e^{\alpha_n b_k} - B_k e^{-\alpha_n b_k}) = G_{k+1}\left(A_{k+1} e^{\alpha_n b_k} - B_{k+1} e^{-\alpha_n b_k}\right) = \zeta_{\alpha_n,k} \end{cases} \tag{3.21}$$

Build a matrix:

$$H_k(y) = \begin{pmatrix} e^{\alpha_n y} & e^{-\alpha_n y} \\ G_k e^{\alpha_n y} & -G_k e^{-\alpha_n y} \end{pmatrix} \tag{3.22}$$

From the conjugation conditions (3.21), we express A_k, B_k in terms of A_1, B_1[2]

by the recurrent formula:

$$\begin{pmatrix} A_k \\ \\ B_k \end{pmatrix} = -\sum_{m=2}^{N} P_{k,m} \begin{pmatrix} \chi_{\alpha_n,k-1} \\ \\ 0 \end{pmatrix}$$

where $P_{k,m} = H_k^{-1}(b_{k-1}) \cdot H_{k-1}^{-1}(b_{k-1}) \cdot \dots \cdot H_m^{-1}(b_m) H_m(b_{m-1})$
The unknown constance of the first layer are found from the boundary conditions of the problem (3.19).
As a result, analytical representations for movements $W_{\alpha_n,k}$ in transform were obtained. In this work, the case when $N = 3$ is considered.

CHAPTER 4

THE ANTI-PLANE PROBLEM OF THE THEORY OF ELASTICITY FOR THE THREE-LAYER RECTANGULAR REGION WITH INTERFACIAL DEFECTS

4.1. Reducing the initial problem to the one-dimensional problem

From the conjugation conditions (3.21), A_2, B_2, A_3, B_3 were expressed in terms of A_1, B_1 [2]:

$$A_2 = \frac{\left(A_1 G_{21} e^{\alpha_n b_1} + A_1 e^{\alpha_n b_1} + B_1 G_{21} e^{-\alpha_n b_1} - B_1 e^{-\alpha_n b_1} - G_{21}\chi_1\right) e^{-\alpha_n b_1}}{2G_{21}}$$

$$A_3 = \left(\frac{A_1 G_{21} G_{32} e^{2\alpha_n b_1} e^{-\alpha_n b_2} + A_1 G_{21} G_{32} e^{\alpha_n b_2} - A_1 G_{21} e^{2\alpha_n b_1} e^{-\alpha_n b_2} + A_1 G_{21} e^{\alpha_n b_2}}{4G_{21}G_{32}} - \right.$$
$$- \frac{A_1 G_{32} e^{2\alpha_n b_1} e^{-\alpha_n b_2} + A_1 G_{32} e^{\alpha_n b_2} + A_1 e^{2\alpha_n b_1} e^{-\alpha_n b_2} + A_1 e^{\alpha_n b_2} + B_1 G_{21} G_{32} e^{-\alpha_n b_2}}{4G_{21}G_{32}} +$$
$$+ \frac{B_1 G_{21} G_{32} e^{-2\alpha_n b_1} e^{\alpha_n b_2} - B_1 G_{21} e^{-\alpha_n b_2} + B_1 G_{21} e^{-2\alpha_n b_1} e^{\alpha_n b_2} + B_1 G_{32} e^{-\alpha_n b_2}}{4G_{21}G_{32}} -$$
$$- \frac{B_1 G_{32} e^{-2\alpha_n b_1} e^{\alpha_n b_2} - B_1 e^{-\alpha_n b_2} - B_1 e^{-2\alpha_n b_1} e^{\alpha_n b_2} - G_{21} G_{32} \chi_1 e^{\alpha_n b_1} e^{-\alpha_n b_2}}{4G_{21}G_{32}} -$$
$$\left. - \frac{G_{21} G_{32} \chi_1 e^{-\alpha_n b_1} e^{\alpha_n b_2} - 2G_{21} G_{32} \chi_2 + G_{21} \chi_1 e^{\alpha_n b_1} e^{-\alpha_n b_2} - G_{21} \chi_1 e^{-\alpha_n b_1} e^{\alpha_n b_2}}{4G_{21}G_{32}} \right) e^{-\alpha_n b_2}$$

$$B_2 = \frac{\left(A_1 G_{21} e^{\alpha_n b_1} - A_1 e^{\alpha_n b_1} + B_1 G_{21} e^{-\alpha_n b_1} + B_1 e^{-\alpha_n b_1} - G_{21}\chi_1\right) e^{\alpha_n b_1}}{2G_{21}}$$

$$B_3 = \left(\frac{A_1 G_{21} G_{32} e^{2\alpha_n b_1} e^{-\alpha_n b_2} + A_1 G_{21} G_{32} e^{\alpha_n b_2} + A_1 G_{21} e^{2\alpha_n b_1} e^{-\alpha_n b_2} - A_1 G_{21} e^{\alpha_n b_2}}{4 G_{21} G_{32}} - \right.$$
$$- \frac{A_1 G_{32} e^{2\alpha_n b_1} e^{-\alpha_n b_2} + A_1 G_{32} e^{\alpha_n b_2} - A_1 e^{2\alpha_n b_1} e^{-\alpha_n b_2} - A_1 e^{\alpha_n b_2} + B_1 G_{21} G_{32} e^{-\alpha_n b_2}}{4 G_{21} G_{32}} +$$
$$+ \frac{B_1 G_{21} G_{32} e^{-2\alpha_n b_1} e^{\alpha_n b_2} + B_1 G_{21} e^{-\alpha_n b_2} - B_1 G_{21} e^{-2\alpha_n b_1} e^{\alpha_n b_2} + B_1 G_{32} e^{-\alpha_n b_2}}{4 G_{21} G_{32}} -$$
$$- \frac{B_1 G_{32} e^{-2\alpha_n b_1} e^{\alpha_n b_2} + B_1 e^{-\alpha_n b_2} + B_1 e^{-2\alpha_n b_1} e^{\alpha_n b_2} - G_{21} G_{32} \chi_1 e^{\alpha_n b_1} e^{-\alpha_n b_2}}{4 G_{21} G_{32}} -$$
$$\left. - \frac{G_{21} G_{32} \chi_1 e^{-\alpha_n b_1} e^{\alpha_n b_2} - 2 G_{21} G_{32} \chi_2 - G_{21} \chi_1 e^{\alpha_n b_1} e^{-\alpha_n b_2} + G_{21} \chi_1 e^{-\alpha_n b_1} e^{\alpha_n b_2}}{4 G_{21} G_{32}} \right) e^{\alpha_n b_2}$$

The unknown constance of the first layer are found from the boundary conditions of the problem (3.19).
It is derived:

$$A_1 = \frac{G_{21} \left(G_3 G_{32} \alpha_n \chi_1 U_5 + 2 G_3 G_{32} \alpha_n \chi_2 \left(u_5 - u_6 \right) - G_3 \alpha_n \chi_1 U_6 + 4 G_{32} p_{\alpha_n} \right)}{G_3 \left(G_{21} G_{32} \alpha_n U_1 - G_{21} \alpha_n U_2 - G_{32} \alpha_n U_3 + \alpha_n U_4 \right)}$$
$$B_1 = \frac{G_{21} \left(G_3 G_{32} \alpha_n \chi_1 U_5 + 2 G_3 G_{32} \alpha_n \chi_2 \left(u_5 - u_6 \right) - G_3 \alpha_n \chi_1 U_6 + 4 G_{32} p_{\alpha_n} \right)}{G_3 \left(G_{21} G_{32} \alpha_n U_1 - G_{21} \alpha_n U_2 - G_{32} \alpha_n U_3 + \alpha_n U_4 \right)}$$

$$\begin{array}{lll}
u_1 = u_8 e^{\alpha_n b_1} u_9 & u_7 = u_8 u_{12} u_9 & u_{13} = e^{2\alpha_n b_2} \\
u_2 = u_8 e^{-\alpha_n b_1} & u_8 = e^{\alpha_n b} & u_{14} = u_8 u_9 \\
u_3 = u_{11} e^{\alpha_n b_1} & u_9 = e^{-2\alpha_n b_2} & u_{15} = u_8 u_{10} \\
u_4 = u_{11} e^{-\alpha_n b_1} u_{13} & u_{10} = e^{-2\alpha_n b_1} & u_{16} = u_{11} u_{12} \\
u_5 = u_8 e^{-\alpha_n b_2} & u_{11} = e^{-\alpha_n b} & u_{17} = u_{11} u_{13} \\
u_6 = u_{11} e^{\alpha_n b_2} & u_{12} = e^{2\alpha_n b_1} & u_{18} = u_{11} u_{10} u_{13}
\end{array}$$

$$\begin{aligned}
U_1 &= u_7 + u_8 + u_{14} + u_{15} - u_{16} - u_{17} - u_{11} - u_{18} \\
U_2 &= u_7 + u_8 - u_{14} + u_{15} - u_{16} + u_{17} - u_{11} + u_{18} \\
U_3 &= u_7 + u_8 + u_{14} - u_{15} + u_{16} - u_{17} - u_{11} + u_{18} \\
U_4 &= u_7 + u_8 - u_{14} - u_{15} + u_{16} + u_{17} - u_{11} - u_{18} \\
U_5 &= u_1 + u_2 - u_3 - u_4 \\
U_6 &= u_1 + u_2 - u_3 + u_4
\end{aligned}$$

Substitute A_1,B_1,A_2,B_2,A_3,B_3 in W_k,$W_k'(k = 1,2,3)$ from the system (3.19) and using the boundary conditions of the system (3.19). The solution in the transform domain was obtained:

$$W_{\alpha_n,1} = \frac{G_{21}\chi_1\left((1-G_{32})\left(-T_1+T_2\right)+\left(G_{32}+1\right)\left(T_3-T_4\right)\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)} + \\ + \frac{2G_{32}G_{21}\chi_2\left(T_5-T_6\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)} + \\ + \frac{G_{21}4G_{32}p_{\alpha_n}T_7}{G_3\alpha_n\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)} \quad (4.1)$$

$$W_{\alpha_n,2} = \frac{\chi_1\left((1-G_{32})\left(T_8+T_9\right)+\left(G_{32}+1\right)\left(T_{10}+T_{11}\right)\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)} + \\ + \frac{G_{32}\chi_2\left((1-G_{21})\left(-T_{12}+T_{13}\right)+\left(G_{21}+1\right)\left(T_5-T_6\right)\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)} + \\ + \frac{2G_{32}p_{\alpha_n}\left((1-G_{21})\left(-T_{14}\right)+\left(G_{21}+1\right)T_7\right)}{G_3\alpha_n\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)} \quad (4.2)$$

$$W_{\alpha_n,3} = \frac{2\chi_1\left(T_{10}+T_{11}\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)} + \\ + \frac{\chi_2\left((1-G_{21})\left(T_{15}+T_{16}\right)+\left(G_{21}+1\right)\left(T_{17}+T_{18}\right)\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)} + \\ + \frac{p_{\alpha_n}\left(T_7\left(R_2+1\right)+T_{19}\left(R_4-1\right)+T_{20}\left(R_3-1\right)+T_{21}\left(R_1+1\right)\right)}{G_3\alpha_n\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)} \quad (4.3)$$

where:

$$T_1 = e^{-\alpha_n(-b_1+2b_2+y)} + e^{-\alpha_n(-b_1+2b_2-y)}$$
$$T_2 = e^{-\alpha_n(2b+b_1-2b_2+y)} + e^{-\alpha_n(2b+b_1-2b_2-y)}$$
$$T_3 = e^{-\alpha_n(b_1+y)} + e^{-\alpha_n(b_1-y)}$$
$$T_4 = e^{-\alpha_n(2b-b_1+y)} + e^{-\alpha_n(2b-b_1-y)}$$
$$T_5 = e^{-\alpha_n(b_2+y)} + e^{-\alpha_n(b_2-y)}$$
$$T_6 = e^{-\alpha_n(2b-b_2+y)} + e^{-\alpha_n(2b-b_2-y)}$$
$$T_7 = e^{-\alpha_n(b+y)} + e^{-\alpha_n(b-y)}$$
$$T_8 = e^{-\alpha_n(b_1+2b_2-y)} - e^{-\alpha_n(-b_1+2b_2-y)}$$
$$T_9 = e^{-\alpha_n(2b+b_1-2b_2+y)} - e^{-\alpha_n(2b-b_1-2b_2+y)}$$
$$T_{10} = e^{-\alpha_n(b_1+y)} - e^{-\alpha_n(-b_1+y)}$$
$$T_{11} = e^{-\alpha_n(2b+b_1-y)} - e^{-\alpha_n(2b-b_1-y)}$$

$$
\begin{aligned}
T_{12} &= e^{-\alpha_n(2b_1+b_2-y)} + e^{-\alpha_n(-2b_1+b_2+y)}\\
T_{13} &= e^{-\alpha_n(2b+2b_1-b_2-y)} + e^{-\alpha_n(2b-2b_1-b_2+y)}\\
T_{14} &= e^{-\alpha_n(b+2b_1-y)} + e^{-\alpha_n(b-2b_1+y)}\\
T_{15} &= e^{-\alpha_n(2b_1-b_2+y)} - e^{-\alpha_n(-2b_1+b_2+y)}\\
T_{16} &= e^{-\alpha_n(2b+2b_1-b_2-y)} - e^{-\alpha_n(2b-2b_1+b_2-y)}\\
T_{17} &= e^{-\alpha_n(b_2+y)} - e^{-\alpha_n(-b_2+y)}\\
T_{18} &= e^{-\alpha_n(2b+b_2-y)} - e^{-\alpha_n(2b-b_2-y)}\\
T_{19} &= e^{-\alpha_n(b+2b_1-y)} + e^{-\alpha_n(b-2b_1+y)}\\
T_{20} &= e^{-\alpha_n(b+2b_2-y)} + e^{-\alpha_n(b-2b_2+y)}\\
T_{21} &= e^{-\alpha_n(b+2b_1-2b_2+y)} + e^{-\alpha_n(b-2b_1+2b_2-y)}
\end{aligned}
$$

$$
\begin{aligned}
E_1 &= e^{-2\alpha_n(b_2-b_1)} - e^{-2\alpha_n(b+b_1-b_2)} & R_1 &= G_{21}G_{32} - G_{21} - G_{32}\\
E_2 &= e^{-2\alpha_n b_2} - e^{-2\alpha_n(b-b_2)} & R_2 &= G_{21}G_{32} + G_{21} + G_{32}\\
E_3 &= e^{-2\alpha_n b_1} - e^{-2\alpha_n(b-b_1)} & R_3 &= G_{21}G_{32} - G_{21} + G_{32}\\
& & R_4 &= G_{21}G_{32} + G_{21} - G_{32}
\end{aligned}
$$

$$S = E_1\left(R_1+1\right) + E_2\left(R_3-1\right) + E_3\left(R_4-1\right)$$

4.2. Inversion of integral transforms

The found solutions in the transform domain are inverted according to the formula (3.9). It is derived:

$$
\begin{aligned}
W_1(x,y) = \frac{2}{a}\sum_{n=0}^{\infty}\Bigg[&\frac{G_{21}\chi_1(\xi)\left(\left(1-G_{32}\right)\left(-T_1+T_2\right)+\left(G_{32}+1\right)\left(T_3-T_4\right)\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)}+\\
&+\frac{2G_{32}G_{21}\chi_2(\xi)\left(T_5-T_6\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)}+\\
&+\frac{G_{21}4G_{32}p(\xi)T_7}{G_3\alpha_n\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)}\Bigg]\sin\alpha_n x \qquad (4.4)
\end{aligned}
$$

$$W_2(x,y) = \frac{2}{a}\sum_{n=0}^{\infty}\left[\frac{\chi_1(\xi)\left((1-G_{32})\left(T_8+T_9\right)+\left(G_{32}+1\right)\left(T_{10}+T_{11}\right)\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)}+ \right.$$
$$+\frac{G_{32}\chi_2(\xi)\left((1-G_{21})\left(-T_{12}+T_{13}\right)+\left(G_{21}+1\right)\left(T_5-T_6\right)\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)}+$$
$$\left. + \frac{2G_{32}p(\xi)\left((1-G_{21})\left(-T_{14}\right)+\left(G_{21}+1\right)T_7\right)}{G_3\alpha_n\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)}\right]\sin\alpha_n x \quad (4.5)$$

$$W_3(x,y) = \frac{2}{a}\sum_{n=0}^{\infty}\left[\frac{2\chi_1(\xi)\left(T_{10}+T_{11}\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)}+ \right.$$
$$+\frac{\chi_2(\xi)\left((1-G_{21})\left(T_{15}+T_{16}\right)+\left(G_{21}+1\right)\left(T_{17}+T_{18}\right)\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)}+$$
$$\left. +\frac{p(\xi)\left(T_7\left(R_2+1\right)+T_{19}\left(R_4-1\right)+T_{20}\left(R_3-1\right)+T_{21}\left(R_1+1\right)\right)}{G_3\alpha_n\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)}\right]\sin\alpha_n x$$
$$(4.6)$$

Considering (3.12), (3.17), and (3.18) formulas (4.4),(4.5) and (4.5) can be rewritten in the following form:

$$W_1(x,y) = \frac{2}{a}\int_0^a\sum_{n=0}^{\infty}\left[\frac{G_{21}\chi_1(\xi)\left((1-G_{32})\left(-T_1+T_2\right)+\left(G_{32}+1\right)\left(T_3-T_4\right)\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)}+ \right.$$
$$+\frac{2G_{32}G_{21}\chi_2(\xi)\left(T_5-T_6\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)}+$$
$$\left. +\frac{G_{21}4G_{32}p(\xi)T_7}{G_3\alpha_n\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)}\right]\Omega(x,\xi)\, d\xi \quad (4.7)$$

$$W_2(x,y) = \frac{2}{a}\int_0^a\sum_{n=0}^{\infty}\left[\frac{\chi_1(\xi)\left((1-G_{32})\left(T_8+T_9\right)+\left(G_{32}+1\right)\left(T_{10}+T_{11}\right)\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)}+ \right.$$
$$+\frac{G_{32}\chi_2(\xi)\left((1-G_{21})\left(-T_{12}+T_{13}\right)+\left(G_{21}+1\right)\left(T_5-T_6\right)\right)}{\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)}+$$
$$\left. +\frac{2G_{32}p(\xi)\left((1-G_{21})\left(-T_{14}\right)+\left(G_{21}+1\right)T_7\right)}{G_3\alpha_n\left(\left(1-e^{-2\alpha_n b}\right)\left(R_2+1\right)+S\right)}\right]\Omega(x,\xi)\, d\xi \quad (4.8)$$

$$W_3(x,y) = \frac{2}{a}\int_0^a \sum_{n=0}^{\infty}\left[\frac{2\chi_1(\xi)\,(T_{10}+T_{11})}{((1-e^{-2\alpha_n b})\,(R_2+1)+S)}+\right.$$
$$+\frac{\chi_2(\xi)\,((1-G_{21})\,(T_{15}+T_{16})+(G_{21}+1)\,(T_{17}+T_{18}))}{((1-e^{-2\alpha_n b})\,(R_2+1)+S)}+$$
$$\left.+\frac{p(\xi)\,(T_7\,(R_2+1)+T_{19}\,(R_4-1)+T_{20}\,(R_3-1)+T_{21}\,(R_1+1))}{G_3\alpha_n\,((1-e^{-2\alpha_n b})\,(R_2+1)+S)}\right]\Omega(x,\xi)\,d\xi \tag{4.9}$$

4.3. Checking of the boundary conditions

It should be noted that no difficulties arose during the verification of homogeneous boundary conditions, so we will consider the case of non-homogeneous boundary conditions for moving W_3.

$$\left.\frac{\partial W_3}{\partial y}\right|_{y=b} = \frac{p(x)}{G_3},$$

where

$$\left.\frac{\partial W_3}{\partial y}\right|_{y=b} = \frac{2}{a}\int_0^a \sum_{n=0}^{\infty}\frac{p(\xi)\sin(\alpha_n x)\sin(\alpha_n \xi)}{G_3}\,d\xi$$

In order to check this inhomogeneous boundary condition, some transformations should be made:

Changing the order of integration and summation

$$\frac{2}{a}\sum_{n=0}^{\infty}\left[\int_0^a p(\xi)\sin(\alpha_n\xi)\,d\xi\right]\sin(\alpha_n x)$$

Continue the load $p(x)$ in the original problem in an odd way in order to check the inhomogeneity of the boundary condition because there is no material for $-a < x < 0$. Moreover, the expansion in the Fourier series for the expanded $p(x)$ was used. Loads were select in such a way that $p(0) = 0$; otherwise, there will be a discontinuity of the first kind.

4.4. Summarization of weakly convergent parts of series.

Consider the series $\sum\limits_{n=1}^{\infty} a(n)$, which is weakly convergent. For separation of its weakly convergent part, the following technique [3] is used, namely: The series $\sum\limits_{n=1}^{\infty} a(n)$ is split into two terms $\sum\limits_{n=1}^{\infty} a(n) = \sum\limits_{n=0}^{A} a(n) + \sum\limits_{n=A}^{\infty} a(n)$. In the second obtained series, the function is replaced by its asymptotic representation at $n \to \infty$, after which the term $\sum\limits_{n=0}^{A} \tilde{a}(n)$ is added and subtracted, where $\tilde{a}(n)$ is the asymptotic representation of the function $a(n)$. So:

$$\sum_{n=1}^{\infty} a(k) = \sum_{n=0}^{\infty} \tilde{a}(n) + \sum_{n=0}^{A} \left(a(n) - \tilde{a}(n)\right), \ \ A \to \infty \tag{4.10}$$

The series $\sum\limits_{n=0}^{\infty} \tilde{a}(n)$ included in this representation can be summed using the following [4] formulas:

$$\sum_{n=0}^{\infty} e^{-nt} \sin nx = \frac{1}{2} \frac{\sin x}{\cosh t - \cos x} \tag{4.11}$$

$$\sum_{n=0}^{\infty} e^{-nt} \cos nx = \frac{\operatorname{sh} t}{2(\operatorname{ch} t - \cos x)} - \frac{1}{2} \tag{4.12}$$

After integration of the formula (4.11), it is derived:

$$\sum_{n=1}^{\infty} \frac{1}{n} e^{-nt} \cos nx = -\frac{1}{2} \ln\left(\operatorname{ch} t - \cos x\right) \tag{4.13}$$

4.4.1. Summation of weakly convergent parts of $W_1(x;y)$

$$W_1(x,y) = \frac{2}{a}\int_0^a \sum_{n=0}^{\infty} \left[\frac{G_{21}\chi_1(\xi)\left((1-G_{32})(-T_1+T_2)+(G_{32}+1)(T_3-T_4)\right)}{\left((1-e^{-2\alpha_n b})(R_2+1)+S\right)} + \right.$$
$$+\frac{2G_{32}G_{21}\chi_2(\xi)(T_5-T_6)}{\left((1-e^{-2\alpha_n b})(R_2+1)+S\right)}+$$
$$\left.+\frac{G_{21}4G_{32}p(\xi)T_7}{G_3\alpha_n\left((1-e^{-2\alpha_n b})(R_2+1)+S\right)}\right]\Omega(x,\xi)\,d\xi$$

where:

$$T_1 = e^{-\alpha_n(-b_1+2b_2+y)} + e^{-\alpha_n(-b_1+2b_2-y)}$$
$$T_2 = e^{-\alpha_n(2b+b_1-2b_2+y)} + e^{-\alpha_n(2b+b_1-2b_2-y)}$$
$$T_3 = e^{-\alpha_n(b_1+y)} + e^{-\alpha_n(b_1-y)}$$
$$T_4 = e^{-\alpha_n(2b-b_1+y)} + e^{-\alpha_n(2b-b_1-y)}$$
$$T_5 = e^{-\alpha_n(b_2+y)} + e^{-\alpha_n(b_2-y)}$$
$$T_6 = e^{-\alpha_n(2b-b_2+y)} + e^{-\alpha_n(2b-b_2-y)}$$
$$T_7 = e^{-\alpha_n(b+y)} + e^{-\alpha_n(b-y)}$$
$$\Omega(x,\xi) = \frac{1}{2}(\cos\alpha_n(\xi-x) - \cos\alpha_n(\xi+x))$$

Applying the formula (4.12) to the first term, it is obtained:

$$C_0 = -\frac{\operatorname{sh} t_0}{(\operatorname{ch} t_0 - \cos x_0)} \qquad C_7 = -\frac{\operatorname{sh} t_3}{(\operatorname{ch} t_3 - \cos x_1)}$$
$$C_1 = -\frac{\operatorname{sh} t_1}{(\operatorname{ch} t_1 - \cos x_0)} \qquad C_8 = \frac{\operatorname{sh} t_4}{(\operatorname{ch} t_4 - \cos x_0)}$$
$$C_2 = \frac{\operatorname{sh} t_2}{(\operatorname{ch} t_2 - \cos x_0)} \qquad C_9 = \frac{\operatorname{sh} t_5}{(\operatorname{ch} t_5 - \cos x_0)}$$
$$C_3 = \frac{\operatorname{sh} t_3}{(\operatorname{ch} t_3 - \cos x_0)} \qquad C_{10} = -\frac{\operatorname{sh} t_6}{(\operatorname{ch} t_6 - \cos x_0)}$$
$$C_4 = \frac{\operatorname{sh} t_0}{(\operatorname{ch} t_0 - \cos x_1)} \qquad C_{11} = -\frac{\operatorname{sh} t_7}{(\operatorname{ch} t_7 - \cos x_0)}$$
$$C_5 = \frac{\operatorname{sh} t_1}{(\operatorname{ch} t_1 - \cos x_1)} \qquad C_{12} = -\frac{\operatorname{sh} t_4}{(\operatorname{ch} t_4 - \cos x_1)}$$
$$C_6 = -\frac{\operatorname{sh} t_2}{(\operatorname{ch} t_2 - \cos x_1)} \qquad C_{13} = -\frac{\operatorname{sh} t_5}{(\operatorname{ch} t_5 - \cos x_1)}$$

$$C_{14} = \frac{\operatorname{sh} t_6}{(\operatorname{ch} t_6 - \cos x_1)} \qquad C_{15} = \frac{\operatorname{sh} t_7}{(\operatorname{ch} t_7 - \cos x_1)}$$

Where:

$$t_0 = \frac{\pi}{a}(-b_1 + 2b_2 + y); \qquad t_5 = \frac{\pi}{a}(b_1 - y);$$
$$t_1 = \frac{\pi}{a}(-b_1 + 2b_2 - y); \qquad t_6 = \frac{\pi}{a}(2b - b_1 + y);$$
$$t_2 = \frac{\pi}{a}(2b + b_1 - 2b_2 + y); \qquad t_7 = \frac{\pi}{a}(2b - b_1 - y);$$
$$t_3 = \frac{\pi}{a}(2b + b_1 - 2b_2 - y); \qquad x_0 = \frac{\pi}{a}(\xi - x);$$
$$t_4 = \frac{\pi}{a}(b_1 + y); \qquad x_1 = \frac{\pi}{a}(\xi + x);$$

Applying the formula (4.12) to the second term, it is derived:

$$C_{16} = \frac{\operatorname{sh} t_8}{(\operatorname{ch} t_8 - \cos x_0)} \qquad C_{20} = -\frac{\operatorname{sh} t_8}{(\operatorname{ch} t_8 - \cos x_1)}$$
$$C_{17} = \frac{\operatorname{sh} t_9}{(\operatorname{ch} t_9 - \cos x_0)} \qquad C_{21} = -\frac{\operatorname{sh} t_9}{(\operatorname{ch} t_9 - \cos x_1)}$$
$$C_{18} = -\frac{\operatorname{sh} t_{10}}{(\operatorname{ch} t_{10} - \cos x_0)} \qquad C_{22} = \frac{\operatorname{sh} t_{10}}{(\operatorname{ch} t_{10} - \cos x_1)}$$
$$C_{19} = -\frac{\operatorname{sh} t_{11}}{(\operatorname{ch} t_{11} - \cos x_0)} \qquad C_{23} = \frac{\operatorname{sh} t_{11}}{(\operatorname{ch} t_{11} - \cos x_1)}$$

Where:

$$t_8 = \frac{\pi}{a}(b_2 + y); \qquad t_{11} = \frac{\pi}{a}(2b - b_2 - y);$$
$$t_9 = \frac{\pi}{a}(b_2 - y); \qquad x_0 = \frac{\pi}{a}(\xi - x);$$
$$t_{10} = \frac{\pi}{a}(2b - b_2 + y); \qquad x_1 = \frac{\pi}{a}(\xi + x);$$

Applying the formula (4.13) to the third term, it is obtained:

$$C_{24} = -\ln(\operatorname{ch} t_{12} - \cos x_0) \qquad C_{26} = \ln(\operatorname{ch} t_{12} - \cos x_1)$$
$$C_{25} = -\ln(\operatorname{ch} t_{13} - \cos x_0) \qquad C_{27} = \ln(\operatorname{ch} t_{13} - \cos x_1)$$

Where:

$$t_{12} = \frac{\pi}{a}(b+y); \qquad x_0 = \frac{\pi}{a}(\xi - x);$$
$$t_{13} = \frac{\pi}{a}(b-y); \qquad x_1 = \frac{\pi}{a}(\xi + x);$$

Then the weakly convergent part for $W_1(x;y)$ will have the form:

$$\begin{aligned} W_1(x,y) = \frac{2}{a}\int_0^a G_{21}\chi_1(\xi)\left[\frac{1}{2}\sum_{w=0}^{7}\{(1-G_{32})C_w + (1+G_{32})C_{w+8}\} + \right. \\ \left. + \frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_1 - Q_2]\right] d\xi + \\ + \frac{2}{a}\int_0^a 2G_{32}G_{21}\chi_2(\xi)\left[\frac{1}{2}\sum_{w=16}^{23} C_w + \frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_3 - Q_4]\right] d\xi + \\ + \frac{2}{a}\int_0^a 4G_{21}G_{32}p(\xi)\left[\sum_{w=24}^{27}\frac{\pi}{2a}C_w + \frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_5 - Q_6]\right] d\xi \end{aligned} \quad (4.14)$$

here and further:

$$\Xi(x,\xi) = \cos\alpha_n(\xi - x) - \cos\alpha_n(\xi + x)$$
$$Q_1 = \frac{(1-G_{32})(-T_1+T_2) + (G_{32}+1)(T_3-T_4)}{((1-e^{-2\alpha_n b})(R_2+1)+S)}$$
$$Q_2 = \frac{(1-G_{32})(-T_1+T_2) + (G_{32}+1)(T_3-T_4)}{(R_2+1)}$$
$$Q_3 = \frac{T_5 - T_6}{((1-e^{-2\alpha_n b})(R_2+1)+S)}$$
$$Q_4 = \frac{T_5 - T_6}{(R_2+1)}$$
$$Q_5 = \frac{T_7}{G_3\alpha_n((1-e^{-2\alpha_n b})(R_2+1)+S)}$$
$$Q_6 = \frac{T_7}{G_3\alpha_n(R_2+1)}$$

4.4.2. Summation of weakly convergent parts of τ^1_{yz}

$$\frac{\partial W_1}{\partial y}(x;y) = G_1\left[\frac{2}{a}\int_0^a G_{21}\chi_1(\xi)\left[\frac{\alpha}{2}\sum_{w=0}^{7}\left\{(1-G_{32})C_w^{'}+(1+G_{32})C_{w+8}^{'}\right\}+\right.\right.$$
$$\left.+\frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_1^{'}-Q_2^{'}]\right]d\xi+$$
$$+\frac{2}{a}\int_0^a 2G_{32}G_{21}\chi_2(\xi)\left[\frac{\alpha}{2}\sum_{w=16}^{23}C_w^{'}+\frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_3^{'}-Q_4^{'}]\right]d\xi+$$
$$\left.+\frac{2}{a}\int_0^a 4G_{21}G_{32}p(\xi)\left[\frac{\alpha\pi}{2a}\sum_{w=24}^{27}C_w^{'}+\frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_5^{'}-Q_6^{'}]\right]d\xi\right] \quad (4.15)$$

$$C_0^{'}=-\frac{1-\operatorname{ch}t_0\cos x_0}{(\operatorname{ch}t_0-\cos x_0)^2} \qquad C_{12}^{'}=-\frac{1-\operatorname{ch}t_4\cos x_1}{(\operatorname{ch}t_4-\cos x_1)^2}$$
$$C_1^{'}=\frac{1-\operatorname{ch}t_1\cos x_0}{(\operatorname{ch}t_1-\cos x_0)^2} \qquad C_{13}^{'}=\frac{1-\operatorname{ch}t_5\cos x_1}{(\operatorname{ch}t_5-\cos x_1)^2}$$
$$C_2^{'}=\frac{1-\operatorname{ch}t_2\cos x_0}{(\operatorname{ch}t_2-\cos x_0)^2} \qquad C_{14}^{'}=\frac{1-\operatorname{ch}t_6\cos x_1}{(\operatorname{ch}t_6-\cos x_1)^2}$$
$$C_3^{'}=-\frac{1-\operatorname{ch}t_3\cos x_0}{(\operatorname{ch}t_3-\cos x_0)^2} \qquad C_{15}^{'}=-\frac{1-\operatorname{ch}t_7\cos x_1}{(\operatorname{ch}t_7-\cos x_1)^2}$$
$$C_4^{'}=\frac{1-\operatorname{ch}t_0\cos x_1}{(\operatorname{ch}t_0-\cos x_1)^2} \qquad C_{16}^{'}=\frac{1-\operatorname{ch}t_8\cos x_0}{(\operatorname{ch}t_8-\cos x_0)^2}$$
$$C_5^{'}=-\frac{1-\operatorname{ch}t_1\cos x_1}{(\operatorname{ch}t_1-\cos x_1)^2} \qquad C_{17}^{'}=-\frac{1-\operatorname{ch}t_9\cos x_0}{(\operatorname{ch}t_9-\cos x_0)^2}$$
$$C_6^{'}=-\frac{1-\operatorname{ch}t_2\cos x_1}{(\operatorname{ch}t_2-\cos x_1)^2} \qquad C_{18}^{'}=-\frac{1-\operatorname{ch}t_{10}\cos x_0}{(\operatorname{ch}t_{10}-\cos x_0)^2}$$
$$C_7^{'}=\frac{1-\operatorname{ch}t_3\cos x_1}{(\operatorname{ch}t_3-\cos x_1)^2} \qquad C_{19}^{'}=\frac{1-\operatorname{ch}t_{11}\cos x_0}{(\operatorname{ch}t_{11}-\cos x_0)^2}$$
$$C_8^{'}=\frac{1-\operatorname{ch}t_4\cos x_0}{(\operatorname{ch}t_4-\cos x_0)^2} \qquad C_{20}^{'}=-\frac{1-\operatorname{ch}t_8\cos x_1}{(\operatorname{ch}t_8-\cos x_1)^2}$$
$$C_9^{'}=-\frac{1-\operatorname{ch}t_5\cos x_0}{(\operatorname{ch}t_5-\cos x_0)^2} \qquad C_{21}^{'}=\frac{1-\operatorname{ch}t_9\cos x_1}{(\operatorname{ch}t_9-\cos x_1)^2}$$
$$C_{10}^{'}=-\frac{1-\operatorname{ch}t_6\cos x_0}{(\operatorname{ch}t_6-\cos x_0)^2} \qquad C_{22}^{'}=\frac{1-\operatorname{ch}t_{10}\cos x_1}{(\operatorname{ch}t_{10}-\cos x_1)^2}$$
$$C_{11}^{'}=\frac{1-\operatorname{ch}t_7\cos x_0}{(\operatorname{ch}t_7-\cos x_0)^2} \qquad C_{23}^{'}=-\frac{1-\operatorname{ch}t_{11}\cos x_1}{(\operatorname{ch}t_{11}-\cos x_1)^2}$$

$$C'_{24} = -\frac{\operatorname{sh} t_{12}}{(\operatorname{ch} t_{12} - \cos x_0)} \qquad C'_{26} = \frac{\operatorname{sh} t_{12}}{(\operatorname{ch} t_{12} - \cos x_1)}$$

$$C'_{25} = \frac{\operatorname{sh} t_{13}}{(\operatorname{ch} t_{13} - \cos x_0)} \qquad C'_{27} = -\frac{\operatorname{sh} t_{13}}{(\operatorname{ch} t_{13} - \cos x_1)}$$

$$Q'_1 = \frac{(1 - G_{32})\left(-T'_1 + T'_2\right) + (G_{32} + 1)\left(T'_3 - T'_4\right)}{((1 - e^{-2\alpha_n b})(R_2 + 1) + S)}$$

$$Q'_2 = \frac{(1 - G_{32})\left(-T'_1 + T'_2\right) + (G_{32} + 1)\left(T'_3 - T'_4\right)}{(R_2 + 1)}$$

$$Q'_3 = \frac{T'_5 - T'_6}{((1 - e^{-2\alpha_n b})(R_2 + 1) + S)}$$

$$Q'_4 = \frac{T'_5 - T'_6}{(R_2 + 1)}$$

$$Q'_5 = \frac{T'_7}{G_3\alpha_n((1 - e^{-2\alpha_n b})(R_2 + 1) + S)}$$

$$Q'_6 = \frac{T'_7}{G_3\alpha_n(R_2 + 1)}$$

$$T'_1 = -\alpha_n\left(e^{-\alpha_n(-b_1+2b_2+y)} - e^{-\alpha_n(-b_1+2b_2-y)}\right)$$

$$T'_2 = -\alpha_n\left(e^{-\alpha_n(2b+b_1-2b_2+y)} - e^{-\alpha_n(2b+b_1-2b_2-y)}\right)$$

$$T'_3 = -\alpha_n\left(e^{-\alpha_n(b_1+y)} - e^{-\alpha_n(b_1-y)}\right)$$

$$T'_4 = -\alpha_n\left(e^{-\alpha_n(2b-b_1+y)} - e^{-\alpha_n(2b-b_1-y)}\right)$$

$$T'_5 = -\alpha_n\left(e^{-\alpha_n(b_2+y)} - e^{-\alpha_n(b_2-y)}\right)$$

$$T'_6 = -\alpha_n\left(e^{-\alpha_n(2b-b_2+y)} - e^{-\alpha_n(2b-b_2-y)}\right)$$

$$T'_7 = -\alpha_n\left(e^{-\alpha_n(b+y)} - e^{-\alpha_n(b-y)}\right)$$

$$t_0 = \frac{\pi}{a}(-b_1 + 2b_2 + y); \qquad t_8 = \frac{\pi}{a}(b_2 + y);$$

$$t_1 = \frac{\pi}{a}(-b_1 + 2b_2 - y); \qquad t_9 = \frac{\pi}{a}(b_2 - y);$$

$$t_2 = \frac{\pi}{a}(2b + b_1 - 2b_2 + y); \qquad t_{10} = \frac{\pi}{a}(2b - b_2 + y);$$

$$t_3 = \frac{\pi}{a}(2b + b_1 - 2b_2 - y); \qquad t_{11} = \frac{\pi}{a}(2b - b_2 - y);$$

$$t_4 = \frac{\pi}{a}(b_1 + y); \qquad t_{12} = \frac{\pi}{a}(b + y);$$

$$t_5 = \frac{\pi}{a}(b_1 - y); \qquad t_{13} = \frac{\pi}{a}(b - y);$$

$$t_6 = \frac{\pi}{a}(2b - b_1 + y); \qquad x_0 = \frac{\pi}{a}(\xi - x);$$

$$t_7 = \frac{\pi}{a}(2b - b_1 - y); \qquad x_1 = \frac{\pi}{a}(\xi + x);$$

4.4.3. Summation of weakly convergent parts of $W_2(x;y)$

$$W_2(x,y) = \frac{2}{a}\int_0^a \sum_{n=0}^{\infty}\left[\frac{\chi_1(\xi)\left((1-G_{32})\,(T_8+T_9)+(G_{32}+1)\,(T_{10}+T_{11})\right)}{\left((1-e^{-2\alpha_n b})\,(R_2+1)+S\right)}+\right.$$
$$+\frac{G_{32}\chi_2(\xi)\left((1-G_{21})\,(-T_{12}+T_{13})+(G_{21}+1)\,(T_5-T_6)\right)}{\left((1-e^{-2\alpha_n b})\,(R_2+1)+S\right)}+$$
$$\left.+\frac{2G_{32}p(\xi)\left((1-G_{21})\,(-T_{14})+(G_{21}+1)\,T_7\right)}{G_3\alpha_n\left((1-e^{-2\alpha_n b})\,(R_2+1)+S\right)}\right]\Omega(x,\xi)\;d\xi \quad (4.16)$$

where:

$$T_5 = e^{-\alpha_n(b_2+y)} + e^{-\alpha_n(b_2-y)}$$
$$T_6 = e^{-\alpha_n(2b-b_2+y)} + e^{-\alpha_n(2b-b_2-y)}$$
$$T_7 = e^{-\alpha_n(b+y)} + e^{-\alpha_n(b-y)}$$
$$T_8 = e^{-\alpha_n(b_1+2b_2-y)} - e^{-\alpha_n(-b_1+2b_2-y)}$$
$$T_9 = e^{-\alpha_n(2b+b_1-2b_2+y)} - e^{-\alpha_n(2b-b_1-2b_2+y)}$$
$$T_{10} = e^{-\alpha_n(b_1+y)} - e^{-\alpha_n(-b_1+y)}$$
$$T_{11} = e^{-\alpha_n(2b+b_1-y)} - e^{-\alpha_n(2b-b_1-y)}$$
$$T_{12} = e^{-\alpha_n(2b_1+b_2-y)} + e^{-\alpha_n(-2b_1+b_2+y)}$$
$$T_{13} = e^{-\alpha_n(2b+2b_1-b_2-y)} + e^{-\alpha_n(2b-2b_1-b_2+y)}$$
$$T_{14} = e^{-\alpha_n(b+2b_1-y)} + e^{-\alpha_n(b-2b_1+y)}$$
$$\Omega(x,\xi) = \frac{1}{2}(\cos\alpha_n(\xi-x) - \cos\alpha_n(\xi+x))$$

Applying the formula (4.12) to the first term, it is obtained:

$$D_0 = \frac{\operatorname{sh}\vartheta_0}{(\operatorname{ch}\vartheta_0 - \cos x_0)} \qquad D_5 = -\frac{\operatorname{sh}\vartheta_1}{(\operatorname{ch}\vartheta_1 - \cos x_1)}$$
$$D_1 = \frac{\operatorname{sh}\vartheta_1}{(\operatorname{ch}\vartheta_1 - \cos x_0)} \qquad D_6 = -\frac{\operatorname{sh}\vartheta_2}{(\operatorname{ch}\vartheta_2 - \cos x_1)}$$
$$D_2 = \frac{\operatorname{sh}\vartheta_2}{(\operatorname{ch}\vartheta_2 - \cos x_0)} \qquad D_7 = -\frac{\operatorname{sh}\vartheta_3}{(\operatorname{ch}\vartheta_3 - \cos x_1)}$$
$$D_3 = \frac{\operatorname{sh}\vartheta_3}{(\operatorname{ch}\vartheta_3 - \cos x_0)} \qquad D_8 = \frac{\operatorname{sh}\vartheta_4}{(\operatorname{ch}\vartheta_4 - \cos x_0)}$$
$$D_4 = -\frac{\operatorname{sh}\vartheta_0}{(\operatorname{ch}\vartheta_0 - \cos x_1)} \qquad D_9 = \frac{\operatorname{sh}\vartheta_5}{(\operatorname{ch}\vartheta_5 - \cos x_0)}$$

$$D_{10} = \frac{\operatorname{sh}\vartheta_6}{(\operatorname{ch}\vartheta_6 - \cos x_0)} \qquad D_{13} = -\frac{\operatorname{sh}\vartheta_5}{(\operatorname{ch}\vartheta_5 - \cos x_1)}$$

$$D_{11} = \frac{\operatorname{sh}\vartheta_7}{(\operatorname{ch}\vartheta_7 - \cos x_0)} \qquad D_{14} = -\frac{\operatorname{sh}\vartheta_6}{(\operatorname{ch}\vartheta_6 - \cos x_1)}$$

$$D_{12} = -\frac{\operatorname{sh}\vartheta_4}{(\operatorname{ch}\vartheta_4 - \cos x_1)} \qquad D_{15} = -\frac{\operatorname{sh}\vartheta_7}{(\operatorname{ch}\vartheta_7 - \cos x_1)}$$

Where:

$$\vartheta_0 = \frac{\pi}{a}(b_1 + 2b_2 - y); \qquad \vartheta_5 = \frac{\pi}{a}(b_1 - y);$$

$$\vartheta_1 = \frac{\pi}{a}(-b_1 + 2b_2 - y); \qquad \vartheta_6 = \frac{\pi}{a}(2b - b_1 + y);$$

$$\vartheta_2 = \frac{\pi}{a}(2b + b_1 - 2b_2 + y); \qquad \vartheta_7 = \frac{\pi}{a}(2b - b_1 - y);$$

$$\vartheta_3 = \frac{\pi}{a}(2b + b_1 - 2b_2 - y); \qquad x_0 = \frac{\pi}{a}(\xi - x);$$

$$\vartheta_4 = \frac{\pi}{a}(b_1 + y); \qquad x_1 = \frac{\pi}{a}(\xi + x);$$

Applying the formula (4.12) to the second term, it is obtained:

$$D_{16} = -\frac{\operatorname{sh}\vartheta_8}{(\operatorname{ch}\vartheta_8 - \cos x_0)} \qquad D_{24} = \frac{\operatorname{sh}\vartheta_{12}}{(\operatorname{ch}\vartheta_{12} - \cos x_0)}$$

$$D_{17} = -\frac{\operatorname{sh}\vartheta_9}{(\operatorname{ch}\vartheta_9 - \cos x_0)} \qquad D_{25} = \frac{\operatorname{sh}\vartheta_{13}}{(\operatorname{ch}\vartheta_{13} - \cos x_0)}$$

$$D_{18} = \frac{\operatorname{sh}\vartheta_{10}}{(\operatorname{ch}\vartheta_{10} - \cos x_0)} \qquad D_{26} = -\frac{\operatorname{sh}\vartheta_{14}}{(\operatorname{ch}\vartheta_{14} - \cos x_0)}$$

$$D_{19} = \frac{\operatorname{sh}\vartheta_{11}}{(\operatorname{ch}\vartheta_{11} - \cos x_0)} \qquad D_{27} = -\frac{\operatorname{sh}\vartheta_{15}}{(\operatorname{ch}\vartheta_{15} - \cos x_0)}$$

$$D_{20} = \frac{\operatorname{sh}\vartheta_8}{(\operatorname{ch}\vartheta_8 - \cos x_1)} \qquad D_{28} = -\frac{\operatorname{sh}\vartheta_{12}}{(\operatorname{ch}\vartheta_{12} - \cos x_1)}$$

$$D_{21} = \frac{\operatorname{sh}\vartheta_9}{(\operatorname{ch}\vartheta_9 - \cos x_1)} \qquad D_{29} = -\frac{\operatorname{sh}\vartheta_{13}}{(\operatorname{ch}\vartheta_{13} - \cos x_1)}$$

$$D_{22} = -\frac{\operatorname{sh}\vartheta_{10}}{(\operatorname{ch}\vartheta_{10} - \cos x_1)} \qquad D_{30} = \frac{\operatorname{sh}\vartheta_{14}}{(\operatorname{ch}\vartheta_{14} - \cos x_1)}$$

$$D_{23} = -\frac{\operatorname{sh}\vartheta_{11}}{(\operatorname{ch}\vartheta_{11} - \cos x_1)} \qquad D_{31} = \frac{\operatorname{sh}\vartheta_{15}}{(\operatorname{ch}\vartheta_{15} - \cos x_1)}$$

Where:

$$\vartheta_8 = \frac{\pi}{a}(2b_1 + b_2 - y); \qquad \vartheta_{13} = \frac{\pi}{a}(b_2 - y);$$
$$\vartheta_9 = \frac{\pi}{a}(-2b_1 + b_2 + y); \qquad \vartheta_{14} = \frac{\pi}{a}(2b - b_2 + y);$$
$$\vartheta_{10} = \frac{\pi}{a}(2b + 2b_1 - b_2 - y); \qquad \vartheta_{15} = \frac{\pi}{a}(2b - b_2 - y);$$
$$\vartheta_{11} = \frac{\pi}{a}(2b - 2b_1 - b_2 + y); \qquad x_0 = \frac{\pi}{a}(\xi - x);$$
$$\vartheta_{12} = \frac{\pi}{a}(b_2 + y); \qquad x_1 = \frac{\pi}{a}(\xi + x);$$

Applying the formula (4.13) to the third term, it is obtained:

$$D_{32} = \ln(\operatorname{ch}\vartheta_{16} - \cos x_0) \qquad D_{36} = -\ln(\operatorname{ch}\vartheta_{18} - \cos x_0)$$
$$D_{33} = \ln(\operatorname{ch}\vartheta_{17} - \cos x_0) \qquad D_{37} = -\ln(\operatorname{ch}\vartheta_{19} - \cos x_0)$$
$$D_{34} = -\ln(\operatorname{ch}\vartheta_{16} - \cos x_1) \qquad D_{38} = \ln(\operatorname{ch}\vartheta_{18} - \cos x_1)$$
$$D_{35} = -\ln(\operatorname{ch}\vartheta_{17} - \cos x_1) \qquad D_{39} = \ln(\operatorname{ch}\vartheta_{19} - \cos x_1)$$

Where:

$$\vartheta_{16} = \frac{\pi}{a}(b + 2b_1 - y); \qquad \vartheta_{19} = \frac{\pi}{a}(b - y);$$
$$\vartheta_{17} = \frac{\pi}{a}(b - 2b_1 + y); \qquad x_0 = \frac{\pi}{a}(\xi - x);$$
$$\vartheta_{18} = \frac{\pi}{a}(b + y); \qquad x_1 = \frac{\pi}{a}(\xi + x);$$

Then the weakly convergent part for $W_2(x;y)$ will have the form:

$$W_2(x,y) = \frac{2}{a}\int_0^a G_3\chi_1(\xi)\left[\frac{1}{2}\sum_{\kappa=0}^{7}\{(1-G_{32})\,D_\kappa + (1+G_{32})D_{\kappa+8}\} + \right.$$
$$\left. + \frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_7 - Q_8]\right] d\xi +$$
$$+ \frac{2}{a}\int_0^a G_3G_{32}\chi_2(\xi)\left[\frac{1}{2}\sum_{\kappa=16}^{23}\{(1-G_{21})\,D_\kappa + (1+G_{21})D_{\kappa+8}\} + \right.$$
$$\left. + \frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_9 - Q_{10}]\right] d\xi +$$
$$+ \frac{2}{a}\int_0^a 4G_{21}G_{32}p(\xi)\left[\sum_{w=32}^{35}\frac{\pi}{2a}\{(1-G_{21})\,D_\kappa + (1+G_{21})D_{\kappa+4}\} + \right.$$
$$\left. + \frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_{11} - Q_{12}]\right] d\xi \quad (4.17)$$

here and further:

$$\Xi(x,\xi) = \cos\alpha_n(\xi - x) - \cos\alpha_n(\xi + x)$$
$$Q_7 = \frac{(1-G_{32})\,(T_8+T_9) + (G_{32}+1)\,(T_{10}+T_{11})}{((1-e^{-2\alpha_n b})\,(R_2+1)+S)}$$
$$Q_8 = \frac{(1-G_{32})\,(T_8+T_9) + (G_{32}+1)\,(T_{10}+T_{11})}{(R_2+1)}$$
$$Q_9 = \frac{(1-G_{21})\,(-T_{12}+T_{13}) + (G_{21}+1)\,(T_5-T_6)}{((1-e^{-2\alpha_n b})\,(R_2+1)+S)}$$
$$Q_{10} = \frac{(1-G_{21})\,(-T_{12}+T_{13}) + (G_{21}+1)\,(T_5-T_6)}{(R_2+1)}$$
$$Q_{11} = \frac{(1-G_{21})\,(-T_{14}) + (G_{21}+1)\,T_7}{G_3\alpha_n\,((1-e^{-2\alpha_n b})\,(R_2+1)+S)}$$
$$Q_{12} = \frac{(1-G_{21})\,(-T_{14}) + (G_{21}+1)\,T_7}{G_3\alpha_n\,(R_2+1)}$$

4.4.4. Summation of weakly convergent parts of τ_{yz}^2

$$\frac{\partial W_2}{\partial y}(x;y) = G_2 \left[\frac{2}{a} \int_0^a \chi_1(\xi) \left[\frac{\alpha}{2} \sum_{\kappa=0}^{7} \left\{ (1-G_{32})\, D'_\kappa + (1+G_{32}) D'_{\kappa+8} \right\} + \right.\right.$$
$$\left. + \frac{1}{2} \sum_{n=1}^{A} \Xi(x,\xi)[Q'_7 - Q'_8] \right] d\xi +$$
$$+ \frac{2}{a} \int_0^a G_{32}\chi_2(\xi) \left[\frac{\alpha}{2} \sum_{\kappa=16}^{23} \left\{ (1-G_{21})\, D'_\kappa + (1+G_{21}) D'_{\kappa+8} \right\} + \right.$$
$$\left. + \frac{1}{2} \sum_{n=1}^{A} \Xi(x,\xi)[Q'_9 - Q'_{10}] \right] d\xi +$$
$$+ \frac{2}{a} \int_0^a 4G_{21}G_{32}p(\xi) \left[\frac{\alpha\pi}{2a} \sum_{w=32}^{35} \left\{ (1-G_{21})\, D'_\kappa + (1+G_{21}) D'_{\kappa+4} \right\} + \right.$$
$$\left.\left. + \frac{1}{2} \sum_{n=1}^{A} \Xi(x,\xi)[Q'_{11} - Q'_{12}] \right] d\xi \right] \quad (4.18)$$

$$D'_0 = -\frac{1-\operatorname{ch}\vartheta_0 \cos x_0}{(\operatorname{ch}\vartheta_0 - \cos x_0)^2} \qquad D'_{10} = \frac{1-\operatorname{ch}\vartheta_6 \cos x_0}{(\operatorname{ch}\vartheta_6 - \cos x_0)^2}$$

$$D'_1 = -\frac{1-\operatorname{ch}\vartheta_1 \cos x_0}{(\operatorname{ch}\vartheta_1 - \cos x_0)^2} \qquad D'_{11} = -\frac{1-\operatorname{ch}\vartheta_7 \cos x_0}{(\operatorname{ch}\vartheta_7 - \cos x_0)^2}$$

$$D'_2 = \frac{1-\operatorname{ch}\vartheta_2 \cos x_0}{(\operatorname{ch}\vartheta_2 - \cos x_0)^2} \qquad D'_{12} = -\frac{1-\operatorname{ch}\vartheta_4 \cos x_1}{(\operatorname{ch}\vartheta_4 - \cos x_1)^2}$$

$$D'_3 = -\frac{1-\operatorname{ch}\vartheta_3 \cos x_0}{(\operatorname{ch}\vartheta_3 - \cos x_0)^2} \qquad D'_{13} = \frac{1-\operatorname{ch}\vartheta_5 \cos x_1}{(\operatorname{ch}\vartheta_5 - \cos x_1)^2}$$

$$D'_4 = \frac{1-\operatorname{ch}\vartheta_0 \cos x_1}{(\operatorname{ch}\vartheta_0 - \cos x_1)^2} \qquad D'_{14} = -\frac{1-\operatorname{ch}\vartheta_6 \cos x_1}{(\operatorname{ch}\vartheta_6 - \cos x_1)^2}$$

$$D'_5 = \frac{1-\operatorname{ch}\vartheta_1 \cos x_1}{(\operatorname{ch}\vartheta_1 - \cos x_1)^2} \qquad D'_{15} = \frac{1-\operatorname{ch}\vartheta_7 \cos x_1}{(\operatorname{ch}\vartheta_7 - \cos x_1)^2}$$

$$D'_6 = -\frac{1-\operatorname{ch}\vartheta_2 \cos x_1}{(\operatorname{ch}\vartheta_2 - \cos x_1)^2} \qquad D'_{16} = \frac{1-\operatorname{ch}\vartheta_8 \cos x_0}{(\operatorname{ch}\vartheta_8 - \cos x_0)^2}$$

$$D'_7 = \frac{1-\operatorname{ch}\vartheta_3 \cos x_1}{(\operatorname{ch}\vartheta_3 - \cos x_1)^2} \qquad D'_{17} = -\frac{1-\operatorname{ch}\vartheta_9 \cos x_0}{(\operatorname{ch}\vartheta_9 - \cos x_0)^2}$$

$$D'_8 = \frac{1-\operatorname{ch}\vartheta_4 \cos x_0}{(\operatorname{ch}\vartheta_4 - \cos x_0)^2} \qquad D'_{18} = -\frac{1-\operatorname{ch}\vartheta_{10} \cos x_0}{(\operatorname{ch}\vartheta_{10} - \cos x_0)}$$

$$D'_9 = -\frac{1-\operatorname{ch}\vartheta_5 \cos x_0}{(\operatorname{ch}\vartheta_5 - \cos x_0)^2} \qquad D'_{19} = \frac{1-\operatorname{ch}\vartheta_{11} \cos x_0}{(\operatorname{ch}\vartheta_{11} - \cos x_0)^2}$$

$$D'_{20} = -\frac{1-\operatorname{ch}\vartheta_8\cos x_1}{(\operatorname{ch}\vartheta_8-\cos x_1)^2} \qquad D'_{30} = \frac{1-\operatorname{ch}\vartheta_{14}\cos x_1}{(\operatorname{ch}\vartheta_{14}-\cos x_1)^2}$$

$$D'_{21} = \frac{1-\operatorname{ch}\vartheta_9\cos x_1}{(\operatorname{ch}\vartheta_9-\cos x_1)^2} \qquad D'_{31} = -\frac{1-\operatorname{ch}\vartheta_{15}\cos x_1}{(\operatorname{ch}\vartheta_{15}-\cos x_1)^2}$$

$$D'_{22} = \frac{1-\operatorname{ch}\vartheta_{10}\cos x_1}{(\operatorname{ch}\vartheta_{10}-\cos x_1)^2} \qquad D'_{32} = -\frac{\operatorname{sh}\vartheta_{16}}{(\operatorname{ch}\vartheta_{16}-\cos x_0)}$$

$$D'_{23} = -\frac{1-\operatorname{ch}\vartheta_{11}\cos x_1}{(\operatorname{ch}\vartheta_{11}-\cos x_1)^2} \qquad D'_{33} = \frac{\operatorname{sh}\vartheta_{17}}{(\operatorname{ch}\vartheta_{17}-\cos x_0)}$$

$$D'_{24} = \frac{1-\operatorname{ch}\vartheta_{12}\cos x_0}{(\operatorname{ch}\vartheta_{12}-\cos x_0)^2} \qquad D'_{34} = \frac{\operatorname{sh}\vartheta_{16}}{(\operatorname{ch}\vartheta_{16}-\cos x_1)}$$

$$D'_{25} = -\frac{1-\operatorname{ch}\vartheta_{13}\cos x_0}{(\operatorname{ch}\vartheta_{13}-\cos x_0)^2} \qquad D'_{35} = -\frac{\operatorname{sh}\vartheta_{17}}{(\operatorname{ch}\vartheta_{17}-\cos x_1)}$$

$$D'_{26} = -\frac{1-\operatorname{ch}\vartheta_{14}\cos x_0}{(\operatorname{ch}\vartheta_{14}-\cos x_0)^2} \qquad D'_{36} = -\frac{\operatorname{sh}\vartheta_{18}}{(\operatorname{ch}\vartheta_{18}-\cos x_0)}$$

$$D'_{27} = \frac{1-\operatorname{ch}\vartheta_{15}\cos x_0}{(\operatorname{ch}\vartheta_{15}-\cos x_0)^2} \qquad D'_{37} = \frac{\operatorname{sh}\vartheta_{19}}{(\operatorname{ch}\vartheta_{19}-\cos x_0)}$$

$$D'_{28} = -\frac{1-\operatorname{ch}\vartheta_{12}\cos x_1}{(\operatorname{ch}\vartheta_{12}-\cos x_1)^2} \qquad D'_{38} = \frac{\operatorname{sh}\vartheta_{18}}{(\operatorname{ch}\vartheta_{18}-\cos x_1)}$$

$$D'_{29} = \frac{1-\operatorname{ch}\vartheta_{13}\cos x_1}{(\operatorname{ch}\vartheta_{13}-\cos x_1)^2} \qquad D'_{39} = -\frac{\operatorname{sh}\vartheta_{19}}{(\operatorname{ch}\vartheta_{19}-\cos x_1)}$$

$$Q'_7 = \frac{(1-G_{32})\left(T'_8+T'_9\right)+(G_{32}+1)\left(T'_{10}+T'_{11}\right)}{((1-e^{-2\alpha_n b})(R_2+1)+S)}$$

$$Q'_8 = \frac{(1-G_{32})\left(T'_8+T'_9\right)+(G_{32}+1)\left(T'_{10}+T'_{11}\right)}{(R_2+1)}$$

$$Q'_9 = \frac{(1-G_{21})\left(-T'_{12}+T'_{13}\right)+(G_{21}+1)\left(T'_5-T'_6\right)}{((1-e^{-2\alpha_n b})(R_2+1)+S)}$$

$$Q'_{10} = \frac{(1-G_{21})\left(-T'_{12}+T'_{13}\right)+(G_{21}+1)\left(T'_5-T'_6\right)}{(R_2+1)}$$

$$Q'_{11} = \frac{(1-G_{21})\left(-T'_{14}\right)+(G_{21}+1)T'_7}{G_3\alpha_n((1-e^{-2\alpha_n b})(R_2+1)+S)}$$

$$Q'_{12} = \frac{(1-G_{21})\left(-T'_{14}\right)+(G_{21}+1)T'_7}{G_3\alpha_n(R_2+1)}$$

$$T'_5 = -\alpha_n\left(e^{-\alpha_n(b_2+y)}-e^{-\alpha_n(b_2-y)}\right)$$

$$T'_6 = -\alpha_n\left(e^{-\alpha_n(2b-b_2+y)}-e^{-\alpha_n(2b-b_2-y)}\right)$$

$$T'_7 = -\alpha_n\left(e^{-\alpha_n(b+y)}-e^{-\alpha_n(b-y)}\right)$$

$$T'_8 = -\alpha_n\left(-e^{-\alpha_n(b_1+2b_2-y)}+e^{-\alpha_n(-b_1+2b_2-y)}\right)$$

$$T'_9 = -\alpha_n\left(e^{-\alpha_n(2b+b_1-2b_2+y)}-e^{-\alpha_n(2b-b_1-2b_2+y)}\right)$$

$$T_{10}^{'} = -\alpha_n \left(e^{-\alpha_n(b_1+y)} - e^{-\alpha_n(-b_1+y)}\right)$$
$$T_{11}^{'} = -\alpha_n \left(-e^{-\alpha_n(2b+b_1-y)} + e^{-\alpha_n(2b-b_1-y)}\right)$$
$$T_{12}^{'} = -\alpha_n \left(-e^{-\alpha_n(2b_1+b_2-y)} + e^{-\alpha_n(-2b_1+b_2+y)}\right)$$
$$T_{13}^{'} = -\alpha_n \left(-e^{-\alpha_n(2b+2b_1-b_2-y)} + e^{-\alpha_n(2b-2b_1-b_2+y)}\right)$$
$$T_{14}^{'} = -\alpha_n \left(-e^{-\alpha_n(b+2b_1-y)} + e^{-\alpha_n(b-2b_1+y)}\right)$$

$$\vartheta_0 = \frac{\pi}{a}(b_1 + 2b_2 - y);$$
$$\vartheta_1 = \frac{\pi}{a}(-b_1 + 2b_2 - y);$$
$$\vartheta_2 = \frac{\pi}{a}(2b + b_1 - 2b_2 + y);$$
$$\vartheta_3 = \frac{\pi}{a}(2b + b_1 - 2b_2 - y);$$
$$\vartheta_4 = \frac{\pi}{a}(b_1 + y);$$
$$\vartheta_5 = \frac{\pi}{a}(b_1 - y);$$
$$\vartheta_6 = \frac{\pi}{a}(2b - b_1 + y);$$
$$\vartheta_7 = \frac{\pi}{a}(2b - b_1 - y);$$
$$\vartheta_8 = \frac{\pi}{a}(2b_1 + b_2 - y);$$
$$\vartheta_9 = \frac{\pi}{a}(-2b_1 + b_2 + y);$$
$$\vartheta_{10} = \frac{\pi}{a}(2b + 2b_1 - b_2 - y);$$
$$\vartheta_{11} = \frac{\pi}{a}(2b - 2b_1 - b_2 + y);$$
$$\vartheta_{12} = \frac{\pi}{a}(b_2 + y);$$
$$\vartheta_{13} = \frac{\pi}{a}(b_2 - y);$$
$$\vartheta_{14} = \frac{\pi}{a}(2b - b_2 + y);$$
$$\vartheta_{15} = \frac{\pi}{a}(2b - b_2 - y);$$
$$\vartheta_{16} = \frac{\pi}{a}(b + 2b_1 - y);$$
$$\vartheta_{17} = \frac{\pi}{a}(b - 2b_1 + y);$$
$$\vartheta_{18} = \frac{\pi}{a}(b + y);$$
$$\vartheta_{19} = \frac{\pi}{a}(b - y);$$
$$x_0 = \frac{\pi}{a}(\xi - x);$$
$$x_1 = \frac{\pi}{a}(\xi + x);$$

4.4.5. Summation of weakly convergent parts of $W_3(x;y)$

$$W_3(x,y) = \frac{2}{a}\int_0^a \sum_{n=0}^{\infty}\left[\frac{2\chi_1(\xi)\,(T_{10}+T_{11})}{((1-e^{-2\alpha_n b})\,(R_2+1)+S)}+\right.$$
$$+\frac{\chi_2(\xi)\,((1-G_{21})\,(T_{15}+T_{16})+(G_{21}+1)\,(T_{17}+T_{18}))}{((1-e^{-2\alpha_n b})\,(R_2+1)+S)}+$$
$$\left.+\frac{p(\xi)\,(T_7\,(R_2+1)+T_{19}\,(R_4-1)+T_{20}\,(R_3-1)+T_{21}\,(R_1+1))}{G_3\alpha_n\,((1-e^{-2\alpha_n b})\,(R_2+1)+S)}\right]\Omega(x,\xi)\,d\xi$$

where:

$$T_7 = e^{-\alpha_n(b+y)} + e^{-\alpha_n(b-y)}$$
$$T_{10} = e^{-\alpha_n(b_1+y)} - e^{-\alpha_n(-b_1+y)}$$
$$T_{11} = e^{-\alpha_n(2b+b_1-y)} - e^{-\alpha_n(2b-b_1-y)}$$
$$T_{15} = e^{-\alpha_n(2b_1-b_2+y)} - e^{-\alpha_n(-2b_1+b_2+y)}$$
$$T_{16} = e^{-\alpha_n(2b+2b_1-b_2-y)} - e^{-\alpha_n(2b-2b_1+b_2-y)}$$
$$T_{17} = e^{-\alpha_n(b_2+y)} - e^{-\alpha_n(-b_2+y)}$$
$$T_{18} = e^{-\alpha_n(2b+b_2-y)} - e^{-\alpha_n(2b-b_2-y)}$$
$$T_{19} = e^{-\alpha_n(b+2b_1-y)} + e^{-\alpha_n(b-2b_1+y)}$$
$$T_{20} = e^{-\alpha_n(b+2b_2-y)} + e^{-\alpha_n(b-2b_2+y)}$$
$$T_{21} = e^{-\alpha_n(b+2b_1-2b_2+y)} + e^{-\alpha_n(b-2b_1+2b_2-y)}$$
$$\Omega(x,\xi) = \frac{1}{2}(\cos\alpha_n(\xi-x) - \cos\alpha_n(\xi+x))$$

Applying the formula (4.12) to the first term, it is obtained:

$$H_0 = \frac{\operatorname{sh}\upsilon_0}{(\operatorname{ch}\upsilon_0 - \cos x_0)} \qquad H_4 = -\frac{\operatorname{sh}\upsilon_0}{(\operatorname{ch}\upsilon_0 - \cos x_1)}$$
$$H_1 = \frac{\operatorname{sh}\upsilon_1}{(\operatorname{ch}\upsilon_1 - \cos x_0)} \qquad H_5 = -\frac{\operatorname{sh}\upsilon_1}{(\operatorname{ch}\upsilon_1 - \cos x_1)}$$
$$H_2 = \frac{\operatorname{sh}\upsilon_2}{(\operatorname{ch}\upsilon_2 - \cos x_0)} \qquad H_6 = -\frac{\operatorname{sh}\upsilon_2}{(\operatorname{ch}\upsilon_2 - \cos x_1)}$$
$$H_3 = \frac{\operatorname{sh}\upsilon_3}{(\operatorname{ch}\upsilon_3 - \cos x_0)} \qquad H_7 = -\frac{\operatorname{sh}\upsilon_3}{(\operatorname{ch}\upsilon_3 - \cos x_1)}$$

Where:

$$\upsilon_0 = \frac{\pi}{a}(b_1 + y); \qquad \upsilon_3 = \frac{\pi}{a}(2b - b_1 - y);$$
$$\upsilon_1 = \frac{\pi}{a}(-b_1 + y); \qquad x_0 = \frac{\pi}{a}(\xi - x);$$
$$\upsilon_2 = \frac{\pi}{a}(2b + b_1 - y); \qquad x_1 = \frac{\pi}{a}(\xi + x);$$

Applying the formula (4.12) to the second term, it is obtained:

$$H_8 = \frac{\operatorname{sh}\upsilon_4}{(\operatorname{ch}\upsilon_4 - \cos x_0)} \qquad H_{16} = \frac{\operatorname{sh}\upsilon_8}{(\operatorname{ch}\upsilon_8 - \cos x_0)}$$
$$H_9 = \frac{\operatorname{sh}\upsilon_5}{(\operatorname{ch}\upsilon_5 - \cos x_0)} \qquad H_{17} = \frac{\operatorname{sh}\upsilon_9}{(\operatorname{ch}\upsilon_9 - \cos x_0)}$$
$$H_{10} = \frac{\operatorname{sh}\upsilon_6}{(\operatorname{ch}\upsilon_6 - \cos x_0)} \qquad H_{18} = \frac{\operatorname{sh}\upsilon_{10}}{(\operatorname{ch}\upsilon_{10} - \cos x_0)}$$
$$H_{11} = \frac{\operatorname{sh}\upsilon_7}{(\operatorname{ch}\upsilon_7 - \cos x_0)} \qquad H_{19} = \frac{\operatorname{sh}\upsilon_{11}}{(\operatorname{ch}\upsilon_{11} - \cos x_0)}$$
$$H_{12} = -\frac{\operatorname{sh}\upsilon_4}{(\operatorname{ch}\upsilon_4 - \cos x_1)} \qquad H_{20} = -\frac{\operatorname{sh}\upsilon_8}{(\operatorname{ch}\upsilon_8 - \cos x_1)}$$
$$H_{13} = -\frac{\operatorname{sh}\upsilon_5}{(\operatorname{ch}\upsilon_5 - \cos x_1)} \qquad H_{21} = -\frac{\operatorname{sh}\upsilon_9}{(\operatorname{ch}\upsilon_9 - \cos x_1)}$$
$$H_{14} = -\frac{\operatorname{sh}\upsilon_6}{(\operatorname{ch}\upsilon_6 - \cos x_1)} \qquad H_{22} = -\frac{\operatorname{sh}\upsilon_{10}}{(\operatorname{ch}\upsilon_{10} - \cos x_1)}$$
$$H_{15} = -\frac{\operatorname{sh}\upsilon_7}{(\operatorname{ch}\upsilon_7 - \cos x_1)} \qquad H_{23} = -\frac{\operatorname{sh}\upsilon_{11}}{(\operatorname{ch}\upsilon_{11} - \cos x_1)}$$

Where:

$$\upsilon_4 = \frac{\pi}{a}(2b_1 - b_2 + y); \qquad \upsilon_9 = \frac{\pi}{a}(-b_2 + y);$$
$$\upsilon_5 = \frac{\pi}{a}(-2b_1 + b_2 + y); \qquad \upsilon_{10} = \frac{\pi}{a}(2b + b_2 - y);$$
$$\upsilon_6 = \frac{\pi}{a}(2b + 2b_1 - b_2 - y); \qquad \upsilon_{11} = \frac{\pi}{a}(2b - b_2 - y);$$
$$\upsilon_7 = \frac{\pi}{a}(2b - 2b_1 + b_2 - y); \qquad x_0 = \frac{\pi}{a}(\xi - x);$$
$$\upsilon_8 = \frac{\pi}{a}(b_2 + y); \qquad x_1 = \frac{\pi}{a}(\xi + x);$$

Applying the formula (4.13) to the third term, it is obtained:

$$H_{24} = -\ln(\operatorname{ch} \upsilon_{12} - \cos x_0) \qquad H_{32} = -\ln(\operatorname{ch} \upsilon_{16} - \cos x_0)$$
$$H_{25} = -\ln(\operatorname{ch} \upsilon_{13} - \cos x_0) \qquad H_{33} = -\ln(\operatorname{ch} \upsilon_{17} - \cos x_0)$$
$$H_{26} = \ln(\operatorname{ch} \upsilon_{12} - \cos x_1) \qquad H_{34} = \ln(\operatorname{ch} \upsilon_{16} - \cos x_1)$$
$$H_{27} = \ln(\operatorname{ch} \upsilon_{13} - \cos x_1) \qquad H_{35} = \ln(\operatorname{ch} \upsilon_{17} - \cos x_1)$$
$$H_{28} = -\ln(\operatorname{ch} \upsilon_{14} - \cos x_0) \qquad H_{36} = -\ln(\operatorname{ch} \upsilon_{18} - \cos x_0)$$
$$H_{29} = -\ln(\operatorname{ch} \upsilon_{15} - \cos x_0) \qquad H_{37} = -\ln(\operatorname{ch} \upsilon_{19} - \cos x_0)$$
$$H_{30} = \ln(\operatorname{ch} \upsilon_{14} - \cos x_1) \qquad H_{38} = \ln(\operatorname{ch} \upsilon_{18} - \cos x_1)$$
$$H_{31} = \ln(\operatorname{ch} \upsilon_{15} - \cos x_1) \qquad H_{39} = \ln(\operatorname{ch} \upsilon_{19} - \cos x_1)$$

Where:

$$\upsilon_{12} = \frac{\pi}{a}(b + y); \qquad \upsilon_{17} = \frac{\pi}{a}(b - 2b_2 + y);$$
$$\upsilon_{13} = \frac{\pi}{a}(b - y); \qquad \upsilon_{18} = \frac{\pi}{a}(b + 2b_1 - 2b_2 + y);$$
$$\upsilon_{14} = \frac{\pi}{a}(b + 2b_1 - y); \qquad \upsilon_{19} = \frac{\pi}{a}(b - 2b_1 + 2b_2 - y);$$
$$\upsilon_{15} = \frac{\pi}{a}(b - 2b_1 + y); \qquad x_0 = \frac{\pi}{a}(\xi - x);$$
$$\upsilon_{16} = \frac{\pi}{a}(b + 2b_2 - y); \qquad x_1 = \frac{\pi}{a}(\xi + x);$$

Then the weakly convergent part for $W_3(x;y)$ will have the form:

$$\begin{aligned} W_3(x,y) = \frac{2}{a}\int_0^a 2\chi_1(\xi)\left[\frac{1}{2}\sum_{w=0}^{7} H_w + \frac{1}{2}\sum_{n=1}^{A} \Xi(x,\xi)[Q_{13} - Q_{14}]\right] d\xi + \\ + \frac{2}{a}\int_0^a \chi_2(\xi)\left[\frac{1}{2}\left\{\sum_{w=8}^{15}(1 - G_{21})H_w + (1 + G_{21})H_{w+8}\right\} + \right. \\ \left. + \frac{1}{2}\sum_{n=1}^{A} \Xi(x,\xi)[Q_{15} - Q_{16}]\right] d\xi + \\ + \frac{2}{a}\int_0^a p(\xi)\left[\sum_{w=24}^{27} \frac{\pi}{2a}\{H_w(R_2 + 1) + H_{w+4}(R_4 - 1) + H_{w+8}(R_3 - 1) + \right. \\ + H_{w+12}(R_1 + 1)\}\, \frac{1}{2}\sum_{n=1}^{A} \Xi(x,\xi)[Q_{17} - Q_{18}] d\xi \end{aligned} \tag{4.19}$$

here and further:

$$\Xi(x,\xi) = \cos\alpha_n(\xi - x) - \cos\alpha_n(\xi + x)$$
$$Q_{13} = \frac{T_{10} + T_{11}}{((1 - e^{-2\alpha_n b})(R_2 + 1) + S)}$$
$$Q_{14} = \frac{T_{10} + T_{11}}{(R_2 + 1)}$$
$$Q_{15} = \frac{(1 - G_{21})(T_{15} + T_{16}) + (G_{21} + 1)(T_{17} + T_{18})}{((1 - e^{-2\alpha_n b})(R_2 + 1) + S)}$$
$$Q_{16} = \frac{(1 - G_{21})(T_{15} + T_{16}) + (G_{21} + 1)(T_{17} + T_{18})}{(R_2 + 1)}$$
$$Q_{17} = \frac{T_7(R_2 + 1) + T_{19}(R_4 - 1) + T_{20}(R_3 - 1) + T_{21}(R_1 + 1)}{G_3\alpha_n((1 - e^{-2\alpha_n b})(R_2 + 1) + S)}$$
$$Q_{18} = \frac{T_7(R_2 + 1) + T_{19}(R_4 - 1) + T_{20}(R_3 - 1) + T_{21}(R_1 + 1)}{G_3\alpha_n(R_2 + 1)}$$

4.4.6. Summation of weakly convergent parts of τ_{yz}^3

$$\begin{aligned}\frac{\partial W_3}{\partial y}(x;y) = G_3 \Bigg[\frac{2}{a}\int_0^a 2\chi_1(\xi)\left[\frac{\alpha}{2}\sum_{w=0}^{7} H_w^{'} + \frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_{13}^{'} - Q_{14}^{'}]\right] d\xi + \\ + \frac{2}{a}\int_0^a \chi_2(\xi)\Bigg[\frac{\alpha_n}{2}\left\{\sum_{w=8}^{15}(1 - G_{21})H_w^{'} + (1 + G_{21})H_{w+8}^{'}\right\} + \\ + \frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_{15}^{'} - Q_{16}^{'}]\Bigg] d\xi + \\ + \frac{2}{a}\int_0^a p(\xi)\Bigg[\sum_{w=24}^{27}\frac{\alpha_n\pi}{2a}\Big\{H_w^{'}(R_2 + 1) + H_{w+4}^{'}(R_4 - 1) + H_{w+8}^{'}(R_3 - 1) + \\ + H_{w+12}^{'}(R_1 + 1)\Big\}\frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_{17}^{'} - Q_{18}^{'}]\Bigg] d\xi\end{aligned} \quad (4.20)$$

$$H_0^{'} = \frac{1 - \operatorname{ch}\upsilon_0\cos x_0}{(\operatorname{ch}\upsilon_0 - \cos x_0)^2} \qquad H_4^{'} = -\frac{1 - \operatorname{ch}\upsilon_0\cos x_1}{(\operatorname{ch}\upsilon_0 - \cos x_1)^2}$$
$$H_1^{'} = \frac{1 - \operatorname{ch}\upsilon_1\cos x_0}{(\operatorname{ch}\upsilon_1 - \cos x_0)^2} \qquad H_5^{'} = -\frac{1 - \operatorname{ch}\upsilon_1\cos x_1}{(\operatorname{ch}\upsilon_1 - \cos x_1)^2}$$
$$H_2^{'} = -\frac{1 - \operatorname{ch}\upsilon_2\cos x_0}{(\operatorname{ch}\upsilon_2 - \cos x_0)^2} \qquad H_6^{'} = \frac{1 - \operatorname{ch}\upsilon_2\cos x_1}{(\operatorname{ch}\upsilon_2 - \cos x_1)^2}$$
$$H_3^{'} = -\frac{1 - \operatorname{ch}\upsilon_3\cos x_0}{(\operatorname{ch}\upsilon_3 - \cos x_0)^2} \qquad H_7^{'} = \frac{1 - \operatorname{ch}\upsilon_3\cos x_1}{(\operatorname{ch}\upsilon_3 - \cos x_1)^2}$$

$$H'_8 = \frac{1 - \operatorname{ch} \upsilon_4 \cos x_0}{(\operatorname{ch} \upsilon_4 - \cos x_0)^2} \qquad H'_{24} = -\frac{\operatorname{sh} \upsilon_{12}}{(\operatorname{ch} \upsilon_{12} - \cos x_0)}$$

$$H'_9 = \frac{1 - \operatorname{ch} \upsilon_5 \cos x_0}{(\operatorname{ch} \upsilon_5 - \cos x_0)^2} \qquad H'_{25} = \frac{\operatorname{sh} \upsilon_{13}}{(\operatorname{ch} \upsilon_{13} - \cos x_0)}$$

$$H'_{10} = -\frac{1 - \operatorname{ch} \upsilon_6 \cos x_0}{(\operatorname{ch} \upsilon_6 - \cos x_0)^2} \qquad H'_{26} = \frac{\operatorname{sh} \upsilon_{12}}{(\operatorname{ch} \upsilon_{12} - \cos x_1)}$$

$$H'_{11} = -\frac{1 - \operatorname{ch} \upsilon_7 \cos x_0}{(\operatorname{ch} \upsilon_7 - \cos x_0)^2} \qquad H'_{27} = -\frac{\operatorname{sh} \upsilon_{13}}{(\operatorname{ch} \upsilon_{13} - \cos x_1)}$$

$$H'_{12} = -\frac{1 - \operatorname{ch} \upsilon_4 \cos x_1}{(\operatorname{ch} \upsilon_4 - \cos x_1)^2} \qquad H'_{28} = \frac{\operatorname{sh} \upsilon_{14}}{(\operatorname{ch} \upsilon_{14} - \cos x_0)}$$

$$H'_{13} = -\frac{1 - \operatorname{ch} \upsilon_5 \cos x_1}{(\operatorname{ch} \upsilon_5 - \cos x_1)^2} \qquad H'_{29} = -\frac{\operatorname{sh} \upsilon_{15}}{(\operatorname{ch} \upsilon_{15} - \cos x_0)}$$

$$H'_{14} = \frac{1 - \operatorname{ch} \upsilon_6 \cos x_1}{(\operatorname{ch} \upsilon_6 - \cos x_1)^2} \qquad H'_{30} = -\frac{\operatorname{sh} \upsilon_{14}}{(\operatorname{ch} \upsilon_{14} - \cos x_1)}$$

$$H'_{15} = \frac{1 - \operatorname{ch} \upsilon_7 \cos x_1}{(\operatorname{ch} \upsilon_7 - \cos x_1)^2} \qquad H'_{31} = \frac{\operatorname{sh} \upsilon_{15}}{(\operatorname{ch} \upsilon_{15} - \cos x_1)}$$

$$H'_{16} = \frac{1 - \operatorname{ch} \upsilon_8 \cos x_0}{(\operatorname{ch} \upsilon_8 - \cos x_0)^2} \qquad H'_{32} = \frac{\operatorname{sh} \upsilon_{16}}{(\operatorname{ch} \upsilon_{16} - \cos x_0)}$$

$$H'_{17} = \frac{1 - \operatorname{ch} \upsilon_9 \cos x_0}{(\operatorname{ch} \upsilon_9 - \cos x_0)^2} \qquad H'_{33} = -\frac{\operatorname{sh} \upsilon_{17}}{(\operatorname{ch} \upsilon_{17} - \cos x_0)}$$

$$H'_{18} = -\frac{1 - \operatorname{ch} \upsilon_{10} \cos x_0}{(\operatorname{ch} \upsilon_{10} - \cos x_0)^2} \qquad H'_{34} = -\frac{\operatorname{sh} \upsilon_{16}}{(\operatorname{ch} \upsilon_{16} - \cos x_1)}$$

$$H'_{19} = -\frac{1 - \operatorname{ch} \upsilon_{11} \cos x_0}{(\operatorname{ch} \upsilon_{11} - \cos x_0)^2} \qquad H'_{35} = \frac{\operatorname{sh} \upsilon_{17}}{(\operatorname{ch} \upsilon_{17} - \cos x_1)}$$

$$H'_{20} = -\frac{1 - \operatorname{ch} \upsilon_8 \cos x_1}{(\operatorname{ch} \upsilon_8 - \cos x_1)^2} \qquad H'_{36} = -\frac{\operatorname{sh} \upsilon_{18}}{(\operatorname{ch} \upsilon_{18} - \cos x_0)}$$

$$H'_{21} = -\frac{1 - \operatorname{ch} \upsilon_9 \cos x_1}{(\operatorname{ch} \upsilon_9 - \cos x_1)^2} \qquad H'_{37} = \frac{\operatorname{sh} \upsilon_{19}}{(\operatorname{ch} \upsilon_{19} - \cos x_0)}$$

$$H'_{22} = \frac{1 - \operatorname{ch} \upsilon_{10} \cos x_1}{(\operatorname{ch} \upsilon_{10} - \cos x_1)^2} \qquad H'_{38} = \frac{\operatorname{sh} \upsilon_{18}}{(\operatorname{ch} \upsilon_{18} - \cos x_1)}$$

$$H'_{23} = \frac{1 - \operatorname{ch} \upsilon_{11} \cos x_1}{(\operatorname{ch} \upsilon_{11} - \cos x_1)^2} \qquad H'_{39} = -\frac{\operatorname{sh} \upsilon_{19}}{(\operatorname{ch} \upsilon_{19} - \cos x_1)}$$

$$Q'_{13} = \frac{T'_{10} + T'_{11}}{((1 - e^{-2\alpha_n b})\,(R_2 + 1) + S)}$$

$$Q'_{14} = \frac{T'_{10} + T'_{11}}{(R_2 + 1)}$$

$$Q'_{15} = \frac{(1 - G_{21})\left(T'_{15} + T'_{16}\right) + (G_{21} + 1)\left(T'_{17} + T'_{18}\right)}{((1 - e^{-2\alpha_n b})\,(R_2 + 1) + S)}$$

$$Q_{16}^{'} = \frac{(1 - G_{21})\left(T_{15}^{'} + T_{16}^{'}\right) + (G_{21} + 1)\left(T_{17}^{'} + T_{18}^{'}\right)}{(R_2 + 1)}$$

$$Q_{17}^{'} = \frac{T_7^{'}(R_2 + 1) + T_{19}^{'}(R_4 - 1) + T_{20}^{'}(R_3 - 1) + T_{21}^{'}(R_1 + 1)}{G_3\alpha_n\left(\left(1 - e^{-2\alpha_n b}\right)(R_2 + 1) + S\right)}$$

$$Q_{18}^{'} = \frac{T_7^{'}(R_2 + 1) + T_{19}^{'}(R_4 - 1) + T_{20}^{'}(R_3 - 1) + T_{21}^{'}(R_1 + 1)}{G_3\alpha_n(R_2 + 1)}$$

$$T_7^{'} = -\alpha_n\left(e^{-\alpha_n(b+y)} - e^{-\alpha_n(b-y)}\right)$$
$$T_{10}^{'} = -\alpha_n\left(e^{-\alpha_n(b_1+y)} - e^{-\alpha_n(-b_1+y)}\right)$$
$$T_{11}^{'} = -\alpha_n\left(-e^{-\alpha_n(2b+b_1-y)} + e^{-\alpha_n(2b-b_1-y)}\right)$$
$$T_{15}^{'} = -\alpha_n\left(e^{-\alpha_n(2b_1-b_2+y)} - e^{-\alpha_n(-2b_1+b_2+y)}\right)$$
$$T_{16}^{'} = -\alpha_n\left(-e^{-\alpha_n(2b+2b_1-b_2-y)} + e^{-\alpha_n(2b-2b_1+b_2-y)}\right)$$
$$T_{17}^{'} = -\alpha_n\left(e^{-\alpha_n(b_2+y)} - e^{-\alpha_n(-b_2+y)}\right)$$
$$T_{18}^{'} = -\alpha_n\left(-e^{-\alpha_n(2b+b_2-y)} + e^{-\alpha_n(2b-b_2-y)}\right)$$
$$T_{19}^{'} = -\alpha_n\left(-e^{-\alpha_n(b+2b_1-y)} + e^{-\alpha_n(b-2b_1+y)}\right)$$
$$T_{20}^{'} = -\alpha_n\left(-e^{-\alpha_n(b+2b_2-y)} + e^{-\alpha_n(b-2b_2+y)}\right)$$
$$T_{21}^{'} = -\alpha_n\left(e^{-\alpha_n(b+2b_1-2b_2+y)} - e^{-\alpha_n(b-2b_1+2b_2-y)}\right)$$

$$\upsilon_0 = \frac{\pi}{a}(b_1 + y); \qquad \upsilon_{11} = \frac{\pi}{a}(2b - b_2 - y);$$
$$\upsilon_1 = \frac{\pi}{a}(-b_1 + y); \qquad \upsilon_{12} = \frac{\pi}{a}(b + y);$$
$$\upsilon_2 = \frac{\pi}{a}(2b + b_1 - y); \qquad \upsilon_{13} = \frac{\pi}{a}(b - y);$$
$$\upsilon_3 = \frac{\pi}{a}(2b - b_1 - y); \qquad \upsilon_{14} = \frac{\pi}{a}(b + 2b_1 - y);$$
$$\upsilon_4 = \frac{\pi}{a}(2b_1 - b_2 + y); \qquad \upsilon_{15} = \frac{\pi}{a}(b - 2b_1 + y);$$
$$\upsilon_5 = \frac{\pi}{a}(-2b_1 + b_2 + y); \qquad \upsilon_{16} = \frac{\pi}{a}(b + 2b_2 - y);$$
$$\upsilon_6 = \frac{\pi}{a}(2b + 2b_1 - b_2 - y); \qquad \upsilon_{17} = \frac{\pi}{a}(b - 2b_2 + y);$$
$$\upsilon_7 = \frac{\pi}{a}(2b - 2b_1 + b_2 - y); \qquad \upsilon_{18} = \frac{\pi}{a}(b + 2b_1 - 2b_2 + y);$$
$$\upsilon_8 = \frac{\pi}{a}(b_2 + y); \qquad \upsilon_{19} = \frac{\pi}{a}(b - 2b_1 + 2b_2 - y);$$
$$\upsilon_9 = \frac{\pi}{a}(-b_2 + y); \qquad x_0 = \frac{\pi}{a}(\xi - x);$$
$$\upsilon_{10} = \frac{\pi}{a}(2b + b_2 - y); \qquad x_1 = \frac{\pi}{a}(\xi + x);$$

4.5. Singular integrodifferential equations (SIDE)

4.5.1. Singular integrodifferential equation-1 (SIDE-1)

In the formula (4.15) during the calculation, a singularity was revealed when $y = b_1$ and $x = \xi$. Apply *the first remarkable limit*:

$$\lim_{x\to 0} \frac{1-\cos x}{\frac{x^2}{2}} = 1; \lim_{x\to 0} \frac{\operatorname{ch} x - 1}{\frac{x^2}{2}} = 1$$

to the formula (4.15):

$$\lim_{t_5, x_0 \to 0} \frac{1-\operatorname{ch} t_5 \cos x_0}{(\operatorname{ch} t_5 - \cos x_0)^2} \approx \frac{2(t_5^2 x_0^2 - t_5^2 + x_0^2)}{(t_5^2 + x_0^2)^2} \approx [t_1 \to 0] \approx -\frac{2}{x_0^2}$$

That is, the following result was obtained:

$$C_9^{\prime *} \approx -\frac{2}{x_0^2} \tag{4.21}$$

That formula (4.15) takes the form:

$$C_9^{\prime} = -\frac{1-\operatorname{ch} t_5 \cos x_0}{(\operatorname{ch} t_5 - \cos x_0)^2} \pm C_9^{\prime *} = -\frac{1-\operatorname{ch} t_5 \cos x_0}{(\operatorname{ch} t_5 - \cos x_0)^2} \pm \left(-\frac{2}{x_0^2}\right) \tag{4.22}$$

The search for $\chi_1(x)$ will be carried out on the assumption that the edges of the crack are free of load, i.e. $\tau_{yz}^1 \Big|_{y=b_1-0} = 0$ and it is derived:

$$\Upsilon_1 \frac{d^2}{dx^2} \int_{c_0}^{c_1} \ln \frac{1}{|\xi^* - x^*|} \chi_1(x^*) d\xi + \int_{c_0}^{c_1} \chi_1(\xi^*) f(\xi^*, x^*) d\xi^* = r(\xi^*, x^*), c_0 < x^* < c_1 \tag{4.23}$$

where:
$\Upsilon_1 = \dfrac{G_{21}\alpha_n(G_{32}+1)}{a}$

$$f(\xi^*,x^*) = \frac{2G_{21}}{a}\left[\frac{\alpha}{2}\sum_{w=0}^{7}\left\{(1-G_{32})C_w^{'} + (1+G_{32})C_{w+8}^{'}\right\} + \right.$$
$$\left. + \frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_1^{'} - Q_2^{'}]\right]$$

$$r(\xi^*,x^*) = \frac{2}{a}\int_0^a 4G_{21}G_{32}p(\xi)\left[\frac{\alpha_n\pi}{2a}\sum_{w=24}^{27}C_w^{'} + \frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_5^{'} - Q_6^{'}]\right]d\xi$$

The substitution of variables $\xi = \dfrac{2\xi^* - (c_0 + c_1)}{c_1 - c_0}$, $x = \dfrac{2x^* - (c_0 + c_1)}{c_1 - c_0}$, is done in order to get integration intervals [-1;1]. As a result, SIDE-1 (4.23) takes the following form:

$$\Upsilon_1\frac{d^2}{dx^2}\int_{c_0}^{c_1}\ln\frac{1}{|\xi - x|}\tilde{\chi}_1(x)d\xi + \int_{c_0}^{c_1}\tilde{\chi}_1(\xi)\tilde{f}(\xi.x)d\xi^* = \tilde{r}(\xi,x), -1 < x < 1 \tag{4.24}$$

where:

$\tilde{\chi}_1(\xi) = \chi_1\left(\dfrac{(c_1 - c_0)\xi + (c_1 + c_0)}{2}\right)$,

$\tilde{r}(\xi,x) = \dfrac{(c_1 - c_0)^2}{4}\left(\dfrac{(c_1 - c_0)\xi + (c_1 + c_0)}{2}, \dfrac{(c_1 - c_0)x + (c_1 + c_0)}{2}\right)$.

$\tilde{f}(\xi.x) = \dfrac{(c_1 - c_0)^2}{4}f\left(\dfrac{(c_1 - c_0)\xi + (c_1 + c_0)}{2}, \dfrac{(c_1 - c_0)x + (c_1 + c_0)}{2}\right)$

SIDE-1 (4.24) is solved by the method of orthogonal polynomials [6], which allows taking into account the true orders of the solution's singularities at the ends of integration interval.

According to this method [5], the function $\tilde{\chi}(\xi)$ is expanded into a series by Chebyshev polynomials of the second kind:

$$\tilde{\chi}_1(\xi) = \sum_{k=0}^{\infty}F_l\sqrt{1-\xi^2}U_l(\xi), \xi \in [-1;1] \tag{4.25}$$

where:

U_k are Chebyshev polynomials of the second kind.

F_k are unknown constants.

Given (4.25), (4.24) will have the form:

$$\Upsilon_1 \frac{d^2}{dx^2} \int_{c_0}^{c_1} \sum_{l=0}^{\infty} F_l \sqrt{1-\xi^2} U_l(\xi) \ln \frac{1}{|\xi - x|} + \\ + \int_{-1}^{1} \sum_{l=0}^{\infty} F_l \sqrt{1-\xi^2} U_l(\xi) \tilde{f}(\xi,x) d\xi = \tilde{r}(\xi,x) \quad (4.26)$$

After the change of the integration and summation orders, the following form of the equation, it is derived

$$\Upsilon_1 \sum_{l=0}^{\infty} F_l \frac{d^2}{dx^2} \int_{c_0}^{c_1} \sqrt{1-\xi^2} U_l(\xi) \ln \frac{1}{|\xi - x|} + \\ + \sum_{l=0}^{\infty} F_l \int_{-1}^{1} \sqrt{1-\xi^2} U_l(\xi) \tilde{f}(\xi,x) d\xi = \tilde{r}(\xi,x) \quad (4.27)$$

Using the spectral correspondence [6][c.108 Table 1. № 24]:

$$\frac{d^2}{dx^2} \int_{-1}^{1} \ln \frac{1}{|x-\xi|} \sqrt{1-\xi^2} U_l(\xi) d\xi = -\pi(l+1) U_k(x) \quad (4.28)$$

As a result, the following form of the formula (4.27):

$$\Upsilon_1 \sum_{l=0}^{\infty} F_l(-\pi(l+1)U_l(\xi)) + \sum_{l=0}^{\infty} F_l \int_{-1}^{1} \sqrt{1-\xi^2} U_l(\xi) \tilde{f}(\xi,x) d\xi = \tilde{r}(\xi,x) \quad (4.29)$$

Both parts were multiplied (4.29) by $U_m(x)\sqrt{1-x^2}$ and integrated over the variable x on the interval [-1;1]
The following result is derived:

$$\Upsilon_1 \sum_{l=0}^{\infty} F_l(-\pi(l+1)) \Phi_{l,m} + \sum_{l=0}^{\infty} F_l D_{l,m} = \check{r}_m, m \in [0;\infty) \quad (4.30)$$

where:
$\Phi_{l,m} = \int_{-1}^{1} U_m(x) U_l(x) \sqrt{1-x^2} dx$
$\check{r} = \int_{-1}^{1} \tilde{r} U_m(x) \sqrt{1-x^2} dx$

$$D_{l,m} = \int_{-1}^{1} \sqrt{1-x^2} U_m(x) dx \int_{-1}^{1} U_l(x) \sqrt{1-\xi^2} \tilde{f}(\xi,x) d\xi$$
$$||U_l||^2 = \frac{\pi}{2}$$

Taking into account that: $\Phi_{l,m} = \begin{cases} ||U_m(x)||^2, & m = l \\ 0, & m \neq l \end{cases}$

Got the following form of the formula(4.30):

$$\Upsilon_1 F_m(-\pi(l+1))||U_m(x)||^2 + \sum_{l=0}^{\infty} F_l D_{l,m} = \check{r}_m, m \in [0;\infty) \quad (4.31)$$

where: $||U_m(x)||^2 = \frac{\pi}{2}$

Plot the graph of the jump function:

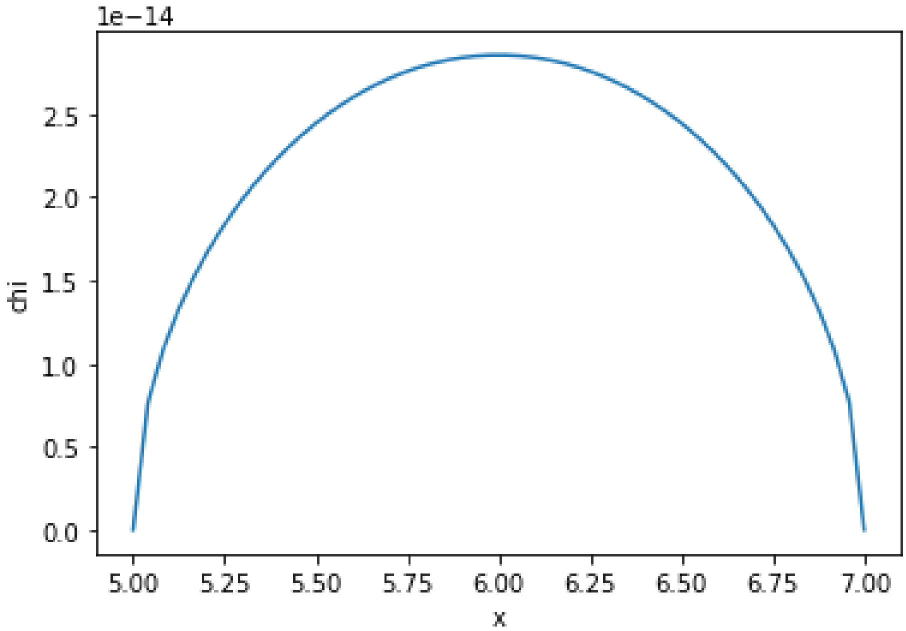


Figure 4.1. The jump function $\chi_1(\xi)$ in the crack

4.5.2. Singular integro-differential equation-2 (SIDE-2)

In the formula (4.18) during the calculation, a singularity was revealed when $y = b_2$ and $x = \xi$. Apply the *first remarkable limits*:

$$\lim_{x\to 0}\frac{1-\cos x}{\frac{x^2}{2}} = 1; \lim_{x\to 0}\frac{\operatorname{ch} x - 1}{\frac{x^2}{2}} = 1$$

to the formula (4.18):

$$\lim_{t_1, x_0\to 0}\frac{1-\operatorname{ch} t_1\cos x_0}{(\operatorname{ch} t_1 - \cos x_0)^2} \approx \frac{2(t_1^2 x_0^2 - t_1^2 + x_0^2)}{(t_1^2 + x_0^2)^2} \approx [t_1 \to 0] \approx -\frac{2}{x_0^2}$$

That is, the following result was obtained:

$$D_{25}^{'*} \approx -\frac{2}{x_0^2} \tag{4.32}$$

That formula (4.18) takes the form:

$$D_{25}^{'} = -\frac{1-\operatorname{ch}\vartheta_{13}\cos x_0}{(\operatorname{ch}\vartheta_{13} - \cos x_0)^2} \pm D_{25}^{'*} = -\frac{1-\operatorname{ch}\vartheta_{13}\cos x_0}{(\operatorname{ch}\vartheta_{13} - \cos x_0)^2} \pm \left(-\frac{2}{x_0^2}\right) \tag{4.33}$$

The search for $\chi_2(x)$ will be carried out on the assumption that the edges of the crack are free of load, i.e. $\tau_{yz}^2\Big|_{y = b_2 - 0} = 0$ and it is derived:

$$\Upsilon_2\frac{d^2}{dx^2}\int_{d_0}^{d_1}\ln\frac{1}{|\xi^* - x^*|}\chi_2(x^*)d\xi + \int_{d_0}^{d_1}\chi_2(\xi^*)g(\xi^*,x^*)d\xi^* = \varrho(\xi^*,x^*), d_0 < x^* < d_1 \tag{4.34}$$

where:

$$\Upsilon_2 = \frac{G_{32}\alpha_n(G_{21}+1)}{a}$$

$$g(\xi^*,x^*) = \frac{2G_{32}}{a}\left[\frac{\alpha}{2}\sum_{\kappa=16}^{23}\left\{(1-G_{21})\,D_{\kappa}^{'} + (1+G_{21})D_{\kappa+8}^{'}\right\} + \right.$$
$$\left. + \frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q_9^{'} - Q_{10}^{'}]\right]$$

$$\varrho(\xi^*,x^*) = \frac{2}{a}\int_0^a 4G_{21}G_{32}p(\xi)\left[\frac{\alpha_n\pi}{2a}\sum_{w=32}^{35}\left\{(1-G_{21})\,D'_{\kappa}+(1+G_{21})D'_{\kappa+4}\right\}+\right.$$
$$\left.+\frac{1}{2}\sum_{n=1}^{A}\Xi(x,\xi)[Q'_{11}-Q'_{12}]\right]d\xi$$

The substitution of variables $\xi = \dfrac{2\xi^*-(d_0+d_1)}{d_1-d_0}$, $x = \dfrac{2x^*-(d_0+d_1)}{d_1-d_0}$, is done in order to get integration intervals [-1;1]. As a result, SIDE-2 (4.23) takes the following form:

$$\Upsilon_2\frac{d^2}{dx^2}\int_{d_0}^{d_1}\ln\frac{1}{|\xi-x|}\tilde{\chi}_2(x)d\xi+\int_{d_0}^{d_1}\tilde{\chi}_2(\xi)\tilde{g}(\xi,x)d\xi^*=\tilde{\varrho}(\xi,x),-1<x<1 \tag{4.35}$$

where:

$\tilde{\chi}_2(\xi) = \chi_2\left(\dfrac{(d_1-d_0)\xi+(d_1+d_0)}{2}\right)$,

$\tilde{\varrho}(\xi,x) = \dfrac{(d_1-d_0)^2}{4}\varrho\left(\dfrac{(d_1-d_0)\xi+(d_1+d_0)}{2},\dfrac{(d_1-d_0)x+(d_1+d_0)}{2}\right)$.

$\tilde{g}(\xi,x) = \dfrac{(d_1-d_0)^2}{4}g\left(\dfrac{(d_1-d_0)\xi+(d_1+d_0)}{2},\dfrac{(d_1-d_0)x+(d_1+d_0)}{2}\right)$

SIDE-2 (4.35) is solved by the method of orthogonal polynomials [6], which allows taking into account the true orders of the solution's singularities at the ends of integration interval. According to this method[5], the function $\tilde{\chi}(\xi)$ is expanded into a series by by Chebyshev polynomials of the second kind:

$$\tilde{\chi}_2(\xi) = \sum_{k=0}^{\infty}O_l\sqrt{1-\xi^2}U_l(\xi),\xi\in[-1;1] \tag{4.36}$$

where:

U_k are Chebyshev polynomials of the second kind.

O_l are unknown constants.

Given (4.36), (4.35) will have the form:

$$\Upsilon_2\frac{d^2}{dx^2}\int_{d_0}^{d_1}\sum_{l=0}^{\infty}O_l\sqrt{1-\xi^2}U_l(\xi)\ln\frac{1}{|\xi-x|}+$$
$$+\int_{-1}^{1}\sum_{l=0}^{\infty}O_l\sqrt{1-\xi^2}U_l(\xi)\tilde{g}(\xi,x)d\xi=\tilde{\varrho}(\xi,x) \tag{4.37}$$

After the change of the integration and summation orders, the following form of the equation, it is derived

$$\Upsilon_2 \sum_{l=0}^{\infty} O_l \frac{d^2}{dx^2} \int_{d_0}^{d_1} \sqrt{1-\xi^2} U_l(\xi) \ln \frac{1}{|\xi - x|} +$$
$$+ \sum_{l=0}^{\infty} O_l \int_{-1}^{1} \sqrt{1-\xi^2} U_l(\xi) \tilde{g}(\xi,x) d\xi = \tilde{\varrho}(\xi,x) \quad (4.38)$$

Using the spectral correspondence [6][c.108 Table 1. № 24]:

$$\frac{d^2}{dx^2} \int_{-1}^{1} \ln \frac{1}{|x-\xi|} \sqrt{1-\xi^2} U_l(\xi) d\xi = -\pi(l+1) U_k(x) \quad (4.39)$$

As a result, the following form of the formula (4.38):

$$\Upsilon_2 \sum_{l=0}^{\infty} O_l(-\pi(l+1)U_l(\xi)) + \sum_{l=0}^{\infty} O_l \int_{-1}^{1} \sqrt{1-\xi^2} U_l(\xi) \tilde{g}(\xi,x) d\xi = \tilde{\varrho}(\xi,x) \quad (4.40)$$

Both parts (4.40) were multiplied by $U_m(x)\sqrt{1-x^2}$ and integrated over the variable x on the interval [-1;1]
The following result is derived

$$\Upsilon_2 \sum_{l=0}^{\infty} O_l(-\pi(l+1)) \Phi_{l,m,2} + \sum_{l=0}^{\infty} O_l D_{l,m,2} = \breve{\varrho}_m, m \in [0; \infty) \quad (4.41)$$

where:

$$\Phi_{l,m,2} = \int_{-1}^{1} U_m(x) U_l(x) \sqrt{1-x^2} dx$$

$$\breve{\varrho} = \int_{-1}^{1} \tilde{\varrho} U_m(x) \sqrt{1-x^2} dx$$

$$D_{l,m,2} = \int_{-1}^{1} \sqrt{1-x^2} U_m(x) dx \int_{-1}^{1} U_l(x) \sqrt{1-\xi^2} \tilde{g}(\xi,x) d\xi$$

$$||U_l||^2 = \frac{\pi}{2}$$

Taking into account that: $\Phi_{l,m,2} = \begin{cases} ||U_m(x)||^2, & m = l \\ 0, & m \neq l \end{cases}$

the following form of the formula (4.41):

$$\Upsilon_2 G_m(-\pi(l+1))||U_m(x)||^2 + \sum_{l=0}^{\infty} O_l D_{l,m,2} = \breve{\varrho}_m, m \in [0;\infty) \qquad (4.42)$$

where: $||U_m(x)||^2 = \dfrac{\pi}{2}$
Plot the graph of the jump function:

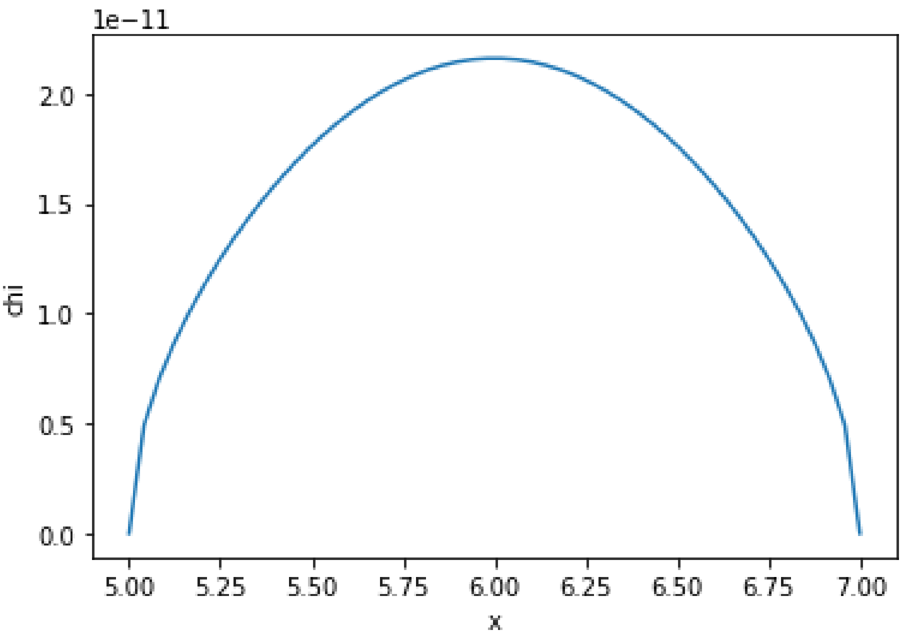


Figure 4.2. The jump function $\chi_2(\xi)$ in a crack

4.6. Stress intensity factors (SIF)

4.6.1. Stress intensity factors-1 (SIF-1)

SIF-1 is calculated according to the formula:

$$K_{\pm} = \lim_{x \to \pm 1 \pm 0} \sqrt{\pi(c_1 - c_0)(\pm x - 1)}\tau_{yz} \tag{4.43}$$

The expression for tangential stress is substituted in the formula (4.43)

$$\frac{1}{\pi}\frac{d^2}{dx^2}\int_{-1}^{1} \ln\frac{1}{|x-\xi|}\sqrt{1-\xi^2}U_l(\xi)d\xi =$$
$$= \frac{|x|U_m(x)}{\sqrt{x^2-1}} + \sqrt{x^2-1}U_m'(x) - 0.5(l+1)U_m(x)$$

and the following spectral correspondence is used.

So, the limit (4.43) takes the form:

$$\lim_{x \to \pm 1 \pm 0} \sqrt{\pi(c_1 - c_0)(\pm x - 1)} \left(\frac{d^2}{dx^2}\int_{-1}^{1} \ln\frac{1}{|x-\xi|}\sqrt{1-\xi^2}U_l(\xi)d\xi\right) =$$
$$= \sqrt{\pi(c_1 - c_0)}(\pm 1)^l(l+1)\frac{1}{\sqrt{2}}$$

As a result, the formulas for the calculation of SIF-1 are derived:

$$K_- = \sum_{l=0}^{\infty} F_k \frac{\sqrt{\pi(c_1 - c_0)}(-1)^l(l+1)}{\sqrt{2}}, K_+ = \sum_{l=0}^{\infty} F_k \frac{\sqrt{\pi(c_1 - c_0)}(l+1)}{\sqrt{2}}$$

4.6.2. SIF-1 when changing longitudinal cracks

Calculate SIF-1 according to the following parameters:

- $a = 12$
- $b = 12$
- $b_1 = 3$
- $b_2 = 9$
- $G_1 = 8.0 \cdot 10^{10}$ — carbon steel
- $G_2 = 4.0 \cdot 10^{10}$ — manganese bronze
- $G_3 = 2.7 \cdot 10^{10}$ — rolled duralumin
- $p(\xi) = \sin \frac{\pi}{a} \xi$

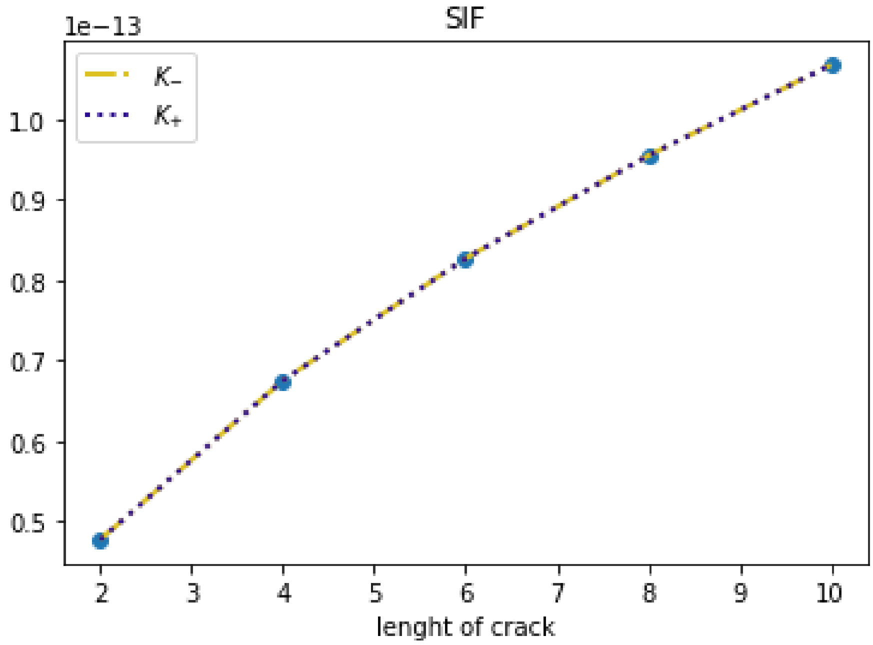


Figure 4.3. Stress intensity factors in the region

Note that the crack is located symmetrically along the x-axis, and its center is located at the point: $\frac{a}{2} = 6$

Analyzing this graph, it can be concluded that the SIF-1 increases when the length of the crack increases.

4.6.3. SIF-1 when the load changes

Calculate the SIF-1 according to the following parameters:

- $a = 12$
- $b = 12$
- $b_1 = 3$
- $b_2 = 9$
- $G_1 = 8.0 \cdot 10^{10}$ — carbon steel
- $G_2 = 4.0 \cdot 10^{10}$ — manganese bronze
- $G_3 = 2.7 \cdot 10^{10}$ — rolled duralumin
- $p(\xi) = \dfrac{2\xi}{a} - \dfrac{4\xi(\xi - \frac{a}{2})}{a^2}$

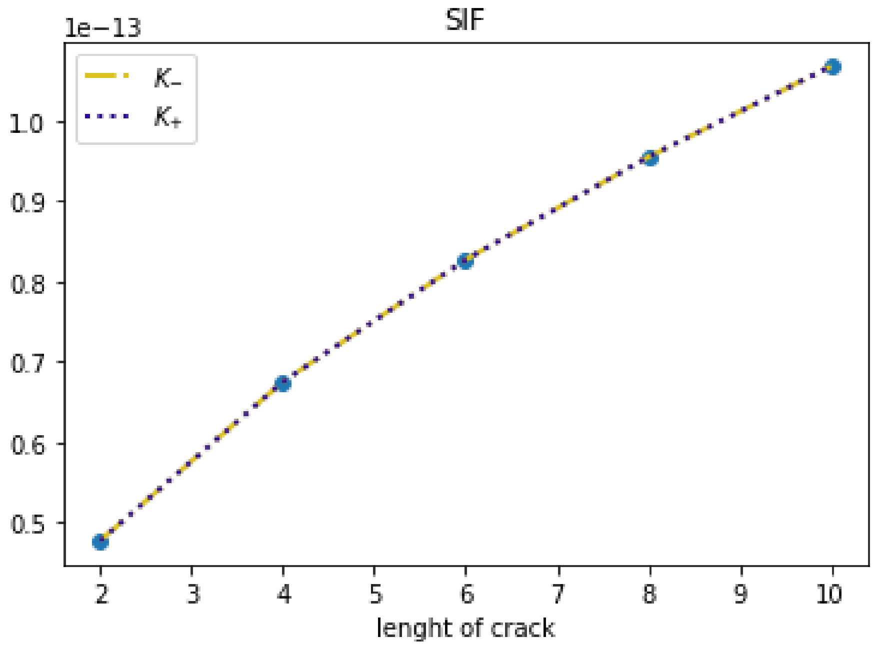


Figure 4.4. Stress intensity factors in the region

Note that the crack is located symmetrically along the x-axis, and its center is located at the point: $\dfrac{a}{2} = 6$

Analyzing this graph, we can conclude that SIF-1 increases when the length of the crack increases.

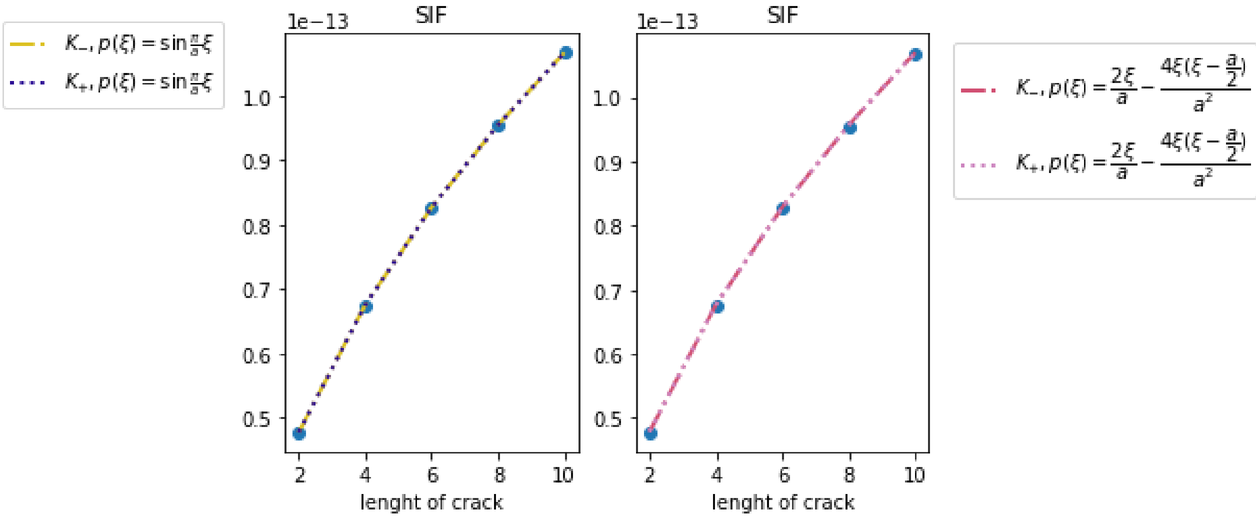


Figure 4.5. Stress intensity factors in the area, depending on the type of load function

Note that the crack is located symmetrically along the x-axis, and its center is located at the point: $\dfrac{a}{2} = 6$

Loads were selected in such a way that $p(0) = 0$.

In the comparative analysis of the types of loads, taking into account the above condition, it was concluded that with this selection of the load function, SIF-1 increases when the length of the crack increases, while the SIF-1 values themselves do not change fundamentally.

4.6.4. Stress intensity factors-2 (SIF-2)

SIF-2 is calculated according to the formula:

$$\Lambda_{\pm} = \lim_{x \to \pm 1 \pm 0} \sqrt{\pi(d_1 - d_0)(\pm x - 1)}\tau_{yz} \tag{4.44}$$

The expression for tangential stress is substituted in the formula (4.44) $\frac{1}{\pi}\frac{d^2}{dx^2}\int_{-1}^{1} \ln\frac{1}{|x-\xi|}\sqrt{1-\xi^2}U_l(\xi)d\xi =$
$= \frac{|x|U_m(x)}{\sqrt{x^2-1}} + \sqrt{x^2-1}U_m^{'}(x) - 0.5(l+1)U_m(x)$ and the following spectral correspondence is used.

So, the limit (4.44) takes the form:

$$\lim_{x \to \pm 1 \pm 0} \sqrt{\pi(d_1 - d_0)(\pm x - 1)}\left(\frac{d^2}{dx^2}\int_{-1}^{1} \ln\frac{1}{|x-\xi|}\sqrt{1-\xi^2}U_l(\xi)d\xi\right) =$$
$$= \sqrt{\pi(d_1 - d_0)}(\pm 1)^l(l+1)\frac{1}{\sqrt{2}}$$

As a result, the formulas for the calculation of SIF-2 are derived:

$$\Lambda_{-} = \sum_{l=0}^{\infty} O_k \frac{\sqrt{\pi(d_1 - d_0)}(-1)^l(l+1)}{\sqrt{2}}, \Lambda_{+} = \sum_{l=0}^{\infty} O_k \frac{\sqrt{\pi(d_1 - d_0)}(l+1)}{\sqrt{2}}$$

4.6.5. SIF-2 when changing longitudinal cracks

Calculate SIF-2 according to the following parameters:

- $a = 12$
- $b = 12$
- $b_1 = 3$
- $b_2 = 9$
- $G_1 = 8.0 \cdot 10^{10}$ — carbon steel
- $G_2 = 4.0 \cdot 10^{10}$ — manganese bronze
- $G_3 = 2.7 \cdot 10^{10}$ — rolled duralumin
- $p(\xi) = \sin \frac{\pi}{a} \xi$

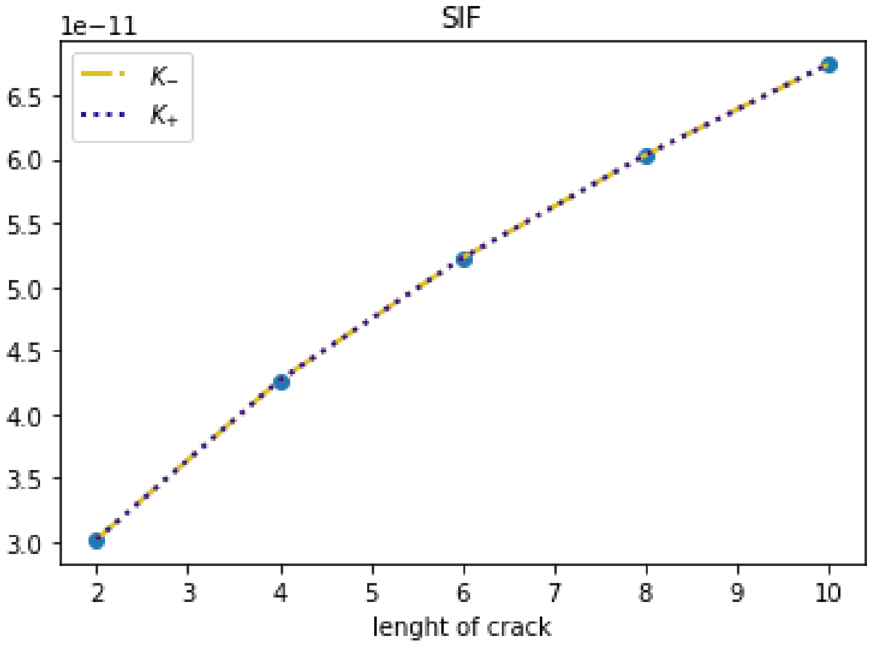


Figure 4.6. Stress intensity factors in the region

Note that the crack is located symmetrically along the x-axis, and its center is located at the point: $\frac{a}{2} = 6$

Analyzing this graph, it can be concluded that the SIF-2 increases when the length of the crack increases.

4.6.6. SIF-2 when the load changes

Calculate the SIF-2 according to the following parameters:

- $a = 12$
- $b = 12$
- $b_1 = 3$
- $b_2 = 9$
- $G_1 = 8.0 \cdot 10^{10}$ — carbon steel
- $G_2 = 4.0 \cdot 10^{10}$ — manganese bronze
- $G_3 = 2.7 \cdot 10^{10}$ — rolled duralumin
- $p(\xi) = \frac{2\xi}{a} - \frac{4\xi(\xi - \frac{a}{2})}{a^2}$

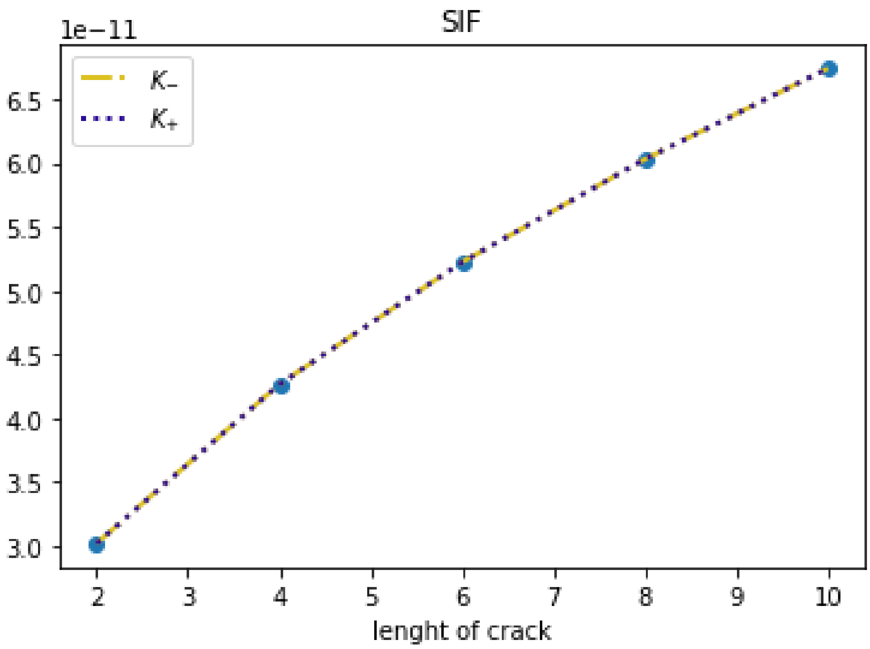


Figure 4.7. Stress intensity factors in the region

Note that the crack is located symmetrically along the x-axis, and its center is located at the point: $\frac{a}{2} = 6$

Analyzing this graph, it can be concluded that the SIF-2 increases when the length of the crack increases.

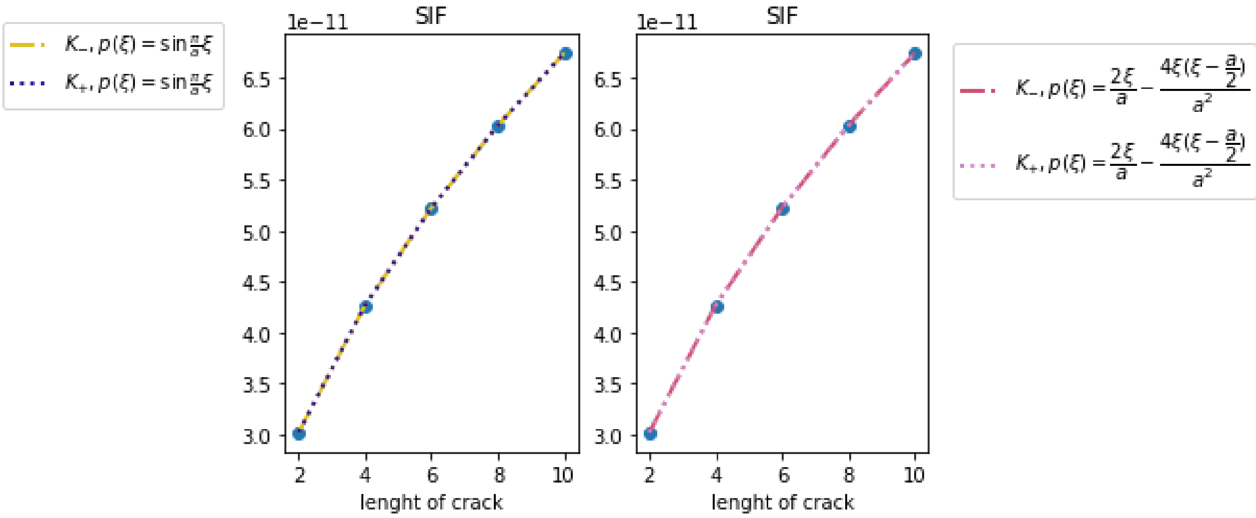


Figure 4.8. Stress intensity factors in the area, depending on the type of load function

Note that the crack is located symmetrically along the x-axis, and its center is located at the point: $\frac{a}{2} = 6$

Loads were selected in such a way that $p(0) = 0$.

In the comparative analysis of the types of loads, taking into account the above condition, it was concluded that with this selection of the load function, SIF-2 increases when the length of the crack increases, while the SIF-2 values themselves do not change fundamentally.

The values of the stress intensity factors and the jump function for this case are greater than for the case when the crack is between the first and second layers, since here the crack is located closer to the loaded edge.

4.7. Conclusions to the fourth section

The investigated anti-plane problem for three-layer rectangular area with an interfacial defect under the influence of various types of loads, which is specified along the y axis.

1) The solution of the anti-plane problem of the theory of elasticity for a rectangular domain is constructed using the apparatus of integral transforms.
2) The change of stress intensity factors during the change of load changes are studied for two cracks.
3) The change of stress intensity factors during the change of crack's length is studied for two cracks

ВИСНОВКИ

В магістерскій роботі досліджено антиплоські задачі для шаруватої прямокутної області під впливом навантаження різної природи, що задано за віссю Oz. Розглянуто задачу для випадку, коли прямокутну область послаблено міжфазною тріщиною. Отримано такі основні результати:

1) Побудован розв'язок антиплоскої задачі теорії пружності для N-шарової прямокутної області з використанням апарату інтегральних перетвореннь. Та окремий випадок, коли область складається з трьох шарів
2) Побудован розв'язок антиплоскої задачі теорії пружності для N-шарової прямокутної області, що послаблена міжфазною тріщиною з використанням апарату інтегральних перетвореннь. Та окремий випадок, коли область складається з трьох шарів
3) Проаналізовані коефіцієнти інтенсивності напружень стосовно довжини тріщини.

CONCLUSIONS

In the master's thesis, anti-plane problems for a layered rectangular area under the influence of a load of different nature along the Oz axis. The problem is considered when the rectangular region is weakened by an interfacial crack. The following main results were obtained:

1) The solution of the anti-plane problem of the theory of elasticity for the N-the layered rectangular region is constructed using the apparatus of integral transformations. The calculations were done for the case when the region consists of three layers
2) The solution of the anti-plane problem of the theory of elasticity for a N-layered rectangular region weakened by an interfacial crack was constructed using the apparatus of integral transformations. The calculations were done for the case when the region consists of three layers
3) The change of stress intensity factors during the change of crack's length is studied for two cracks

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1) Журавльова З.Ю., Чернобровкін А.В. Антиплоска задача теорії пружності для двошарової прямокутної області. //Інформатика, математика, автоматика IMA :: 2020, Матеріали міжнародної науково-технічної конференції студентів та молодих вчених,Суми, Сумський державний університет, 2020– 214-215 с.
2) Журавльова З.Ю., Чернобровкін А.В. Антиплоска задача теорії пружності для двошарової прямокутної області з міжфазною тріщиною. // Інформатика, математика, автоматика IMA :: 2021, Матеріали міжнародної науково-технічної конференції студентів та молодих вчених,Суми, Сумський державний університет, 2021– 200-201 с.
3) Журавльова З.Ю., Чернобровкін А.В. Антиплоска задача теорії пружності для прямокутної області з горизонтальним дефектом. // Інформатика, інформаційні системи та технології: тези доповідей вісімнадцятої всеукраїнської конференції студентів і молодих науковців. – Одеса. – 2021. – 137-138 с.
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12) Zhuravlova Z. Yu., Chernobrovkin A. V. Antiplane elasticity problem for a multilayer rectangular region with interfacial rigid inclusions // The XXV International Scientific and Practical Conference «Innovative trends of science and practice, tasks and ways to solve them», June 28 – July 01, 2022, Athens, Greece. 449-451 pp.

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